Efficient Attention

CS 4804: Introduction to AI

Fall 2025

https://tuvllms.github.io/ai-fall-2025/

Tu Vu

VIRGINIA TECH

Logistics

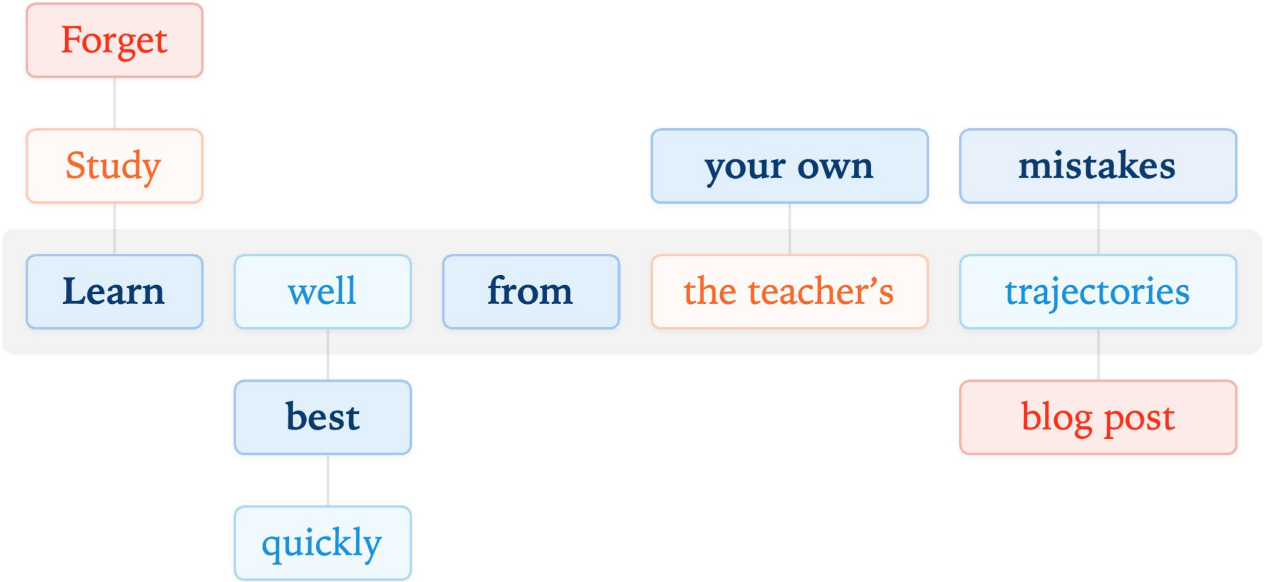
- Feedback for final project proposals today or tomorrow
  - All postponed since last Friday — Tu traveling + sick
  - Same for emails :(
- Quiz 2 due today 10/30
- HW 2 released due 11/18
- Teaching & learning evaluation: 11/4
- Final presentations: 12/4 & 12/9

On-Policy distillation

On-Policy Distillation

Kevin Lu in collaboration with others at Thinking Machines

Oct 27, 2025



On-Policy distillation (cont'd)

- Off-policy training: The student learns by imitating a teacher or dataset of correct answers (like supervised fine-tuning). The student sees what the teacher did, but it doesn't learn directly from its own mistakes.

- On-policy training (reinforcement learning, RL): The student acts (rolls out trajectories) and then gets feedback (reward) based on its own behavior. This aligns the training with what the student actually does, but the feedback is very sparse (you might only know "success/fail" at the end).

On-Policy distillation (cont'd)

The key idea:

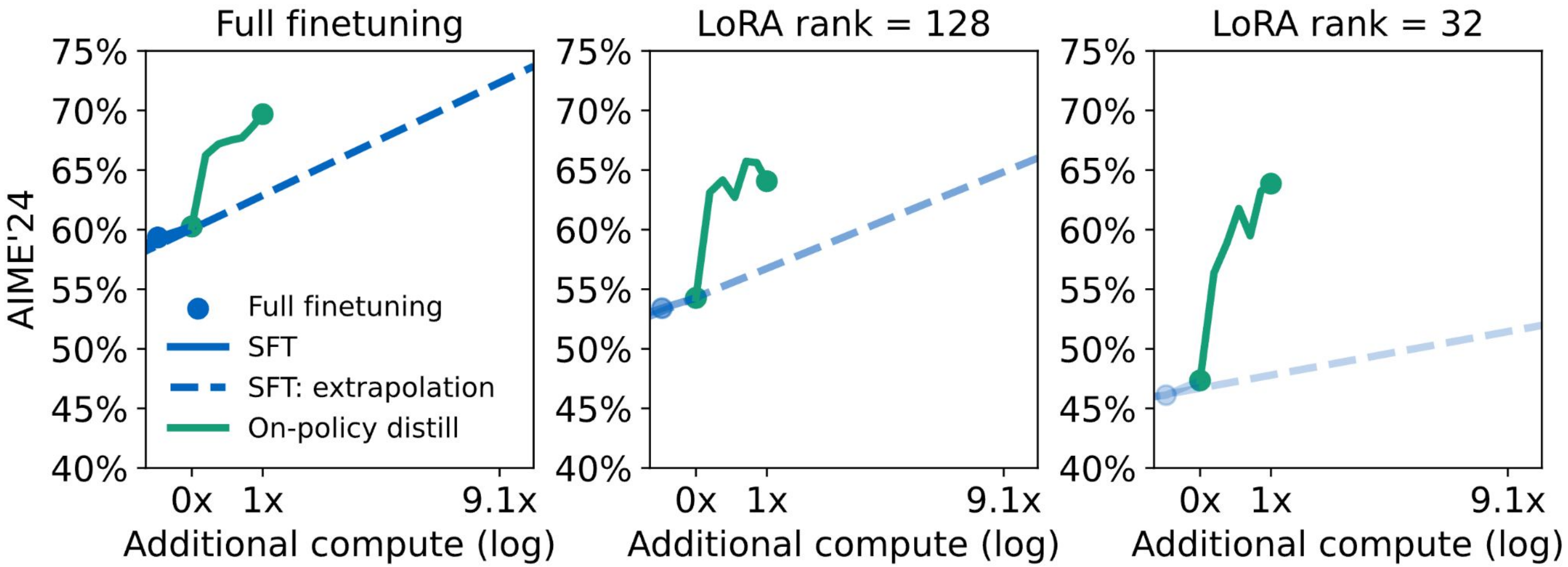
- The student model samples trajectories (i.e., generates outputs) using its own policy (its own behaviour).
- A high-performing teacher model evaluates each token (or each step) of those trajectories: it gives detailed feedback, not just at the end but token-by-token.
- The student then updates itself to minimise the divergence between its behaviour and the teacher's behaviour in the states the student actually visits. In other words, the student learns what the teacher would do when the student is in that situation.

In short: the student learns from its own path, gets dense feedback from the teacher, and thereby merges the benefits of on-policy learning (relevance to its own states) and distillation/imitation (rich feedback) into one.

On-Policy distillation (cont'd)

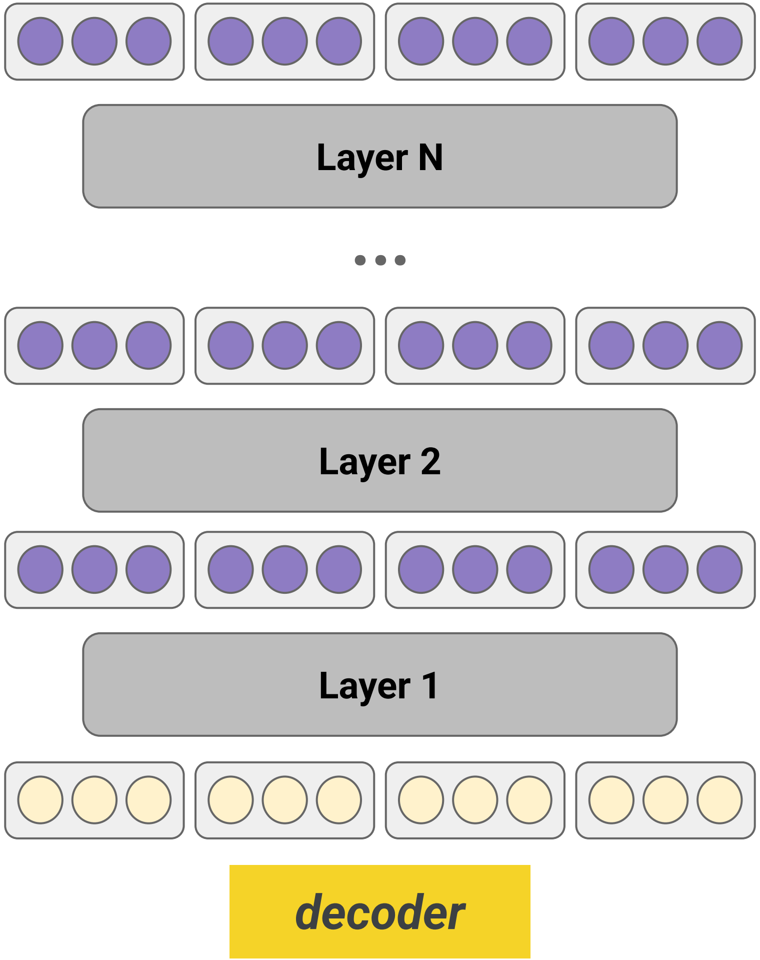
Method	Sampling	Reward signal
Supervised finetuning	off-policy	dense
Reinforcement learning	on-policy	sparse
On-policy distillation	on-policy	dense

Method	AIME'24	GPQA-Diamond	GPU Hours
Off-policy distillation	55.0%	55.6%	Unreported
+ Reinforcement learning	67.6%	61.3%	17,920
+ On-policy distillation	**74.4%**	**63.3%**	**1,800**

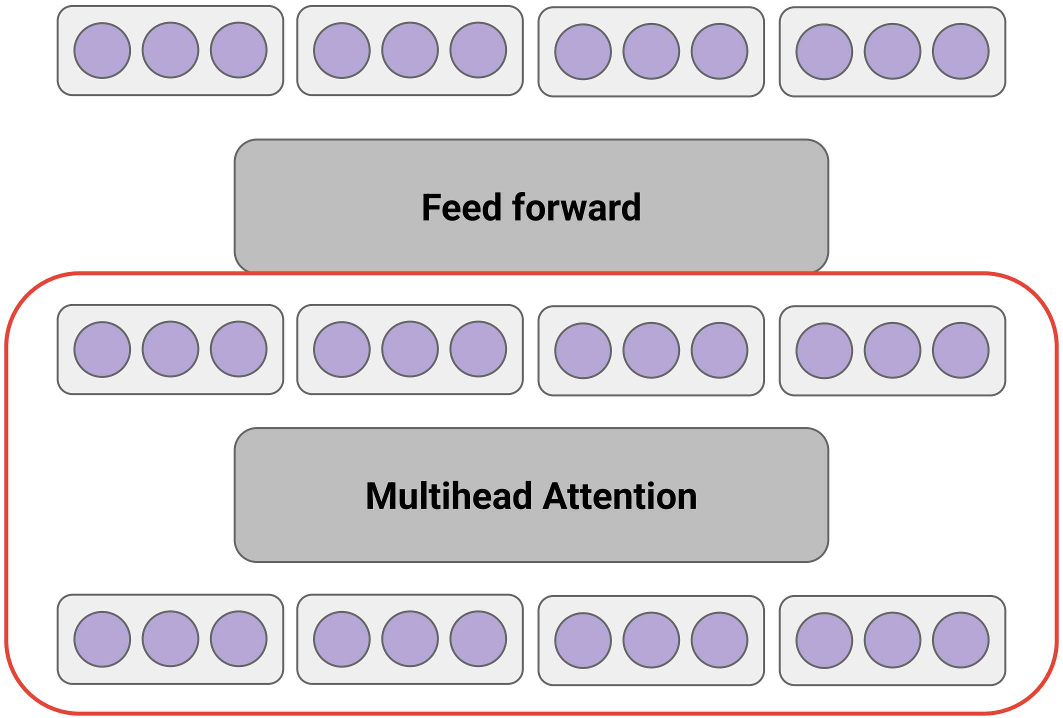


Decoder-only Transformer review

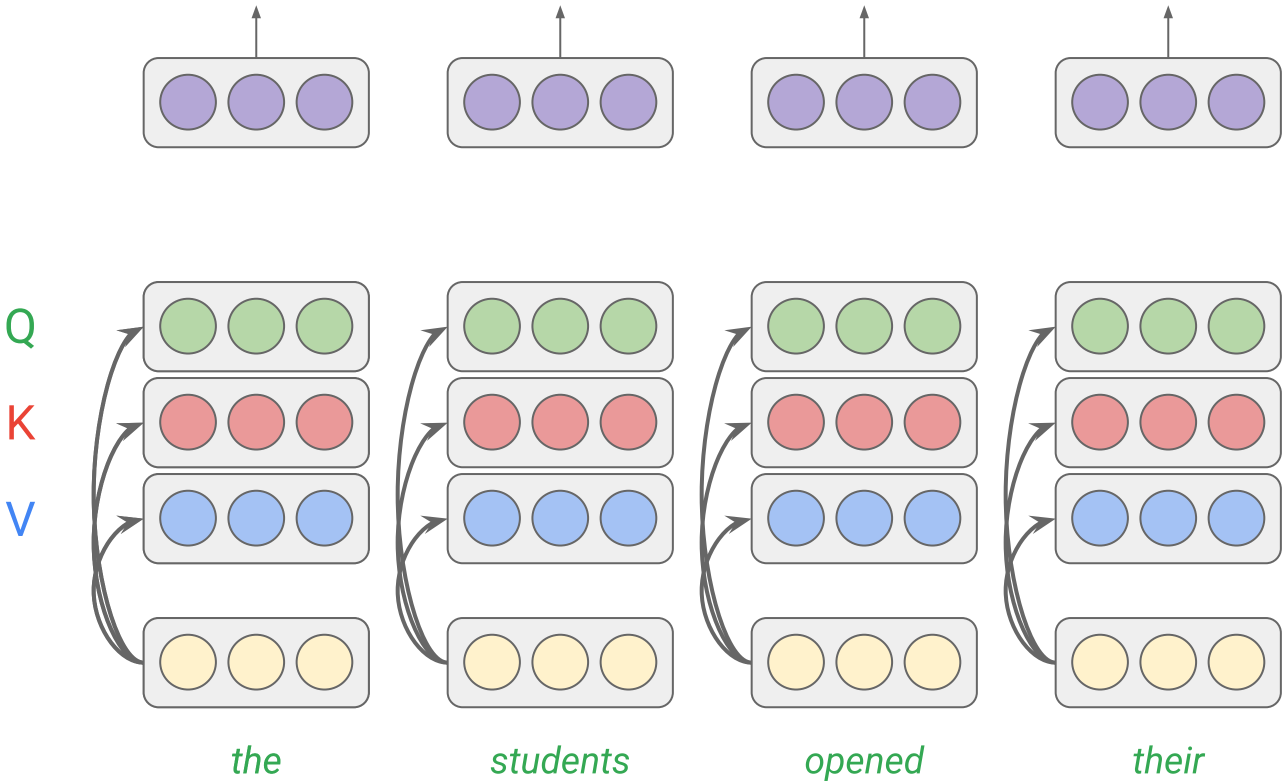
Transformer (N layers)



Transformer decoder



Attention



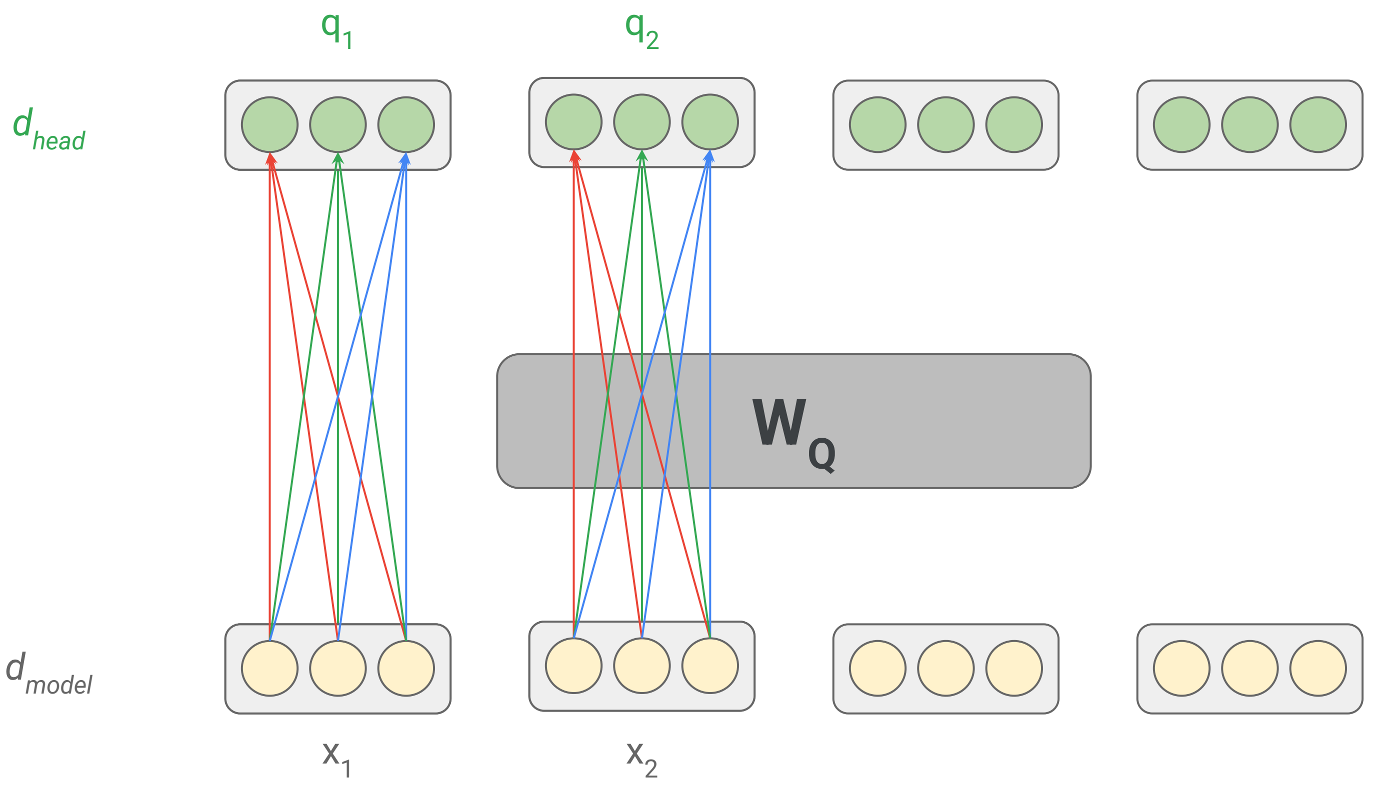
$$Q = X \cdot W_Q$$

$$K = X \cdot W_K$$

$$V = X \cdot W_V$$

linear projections

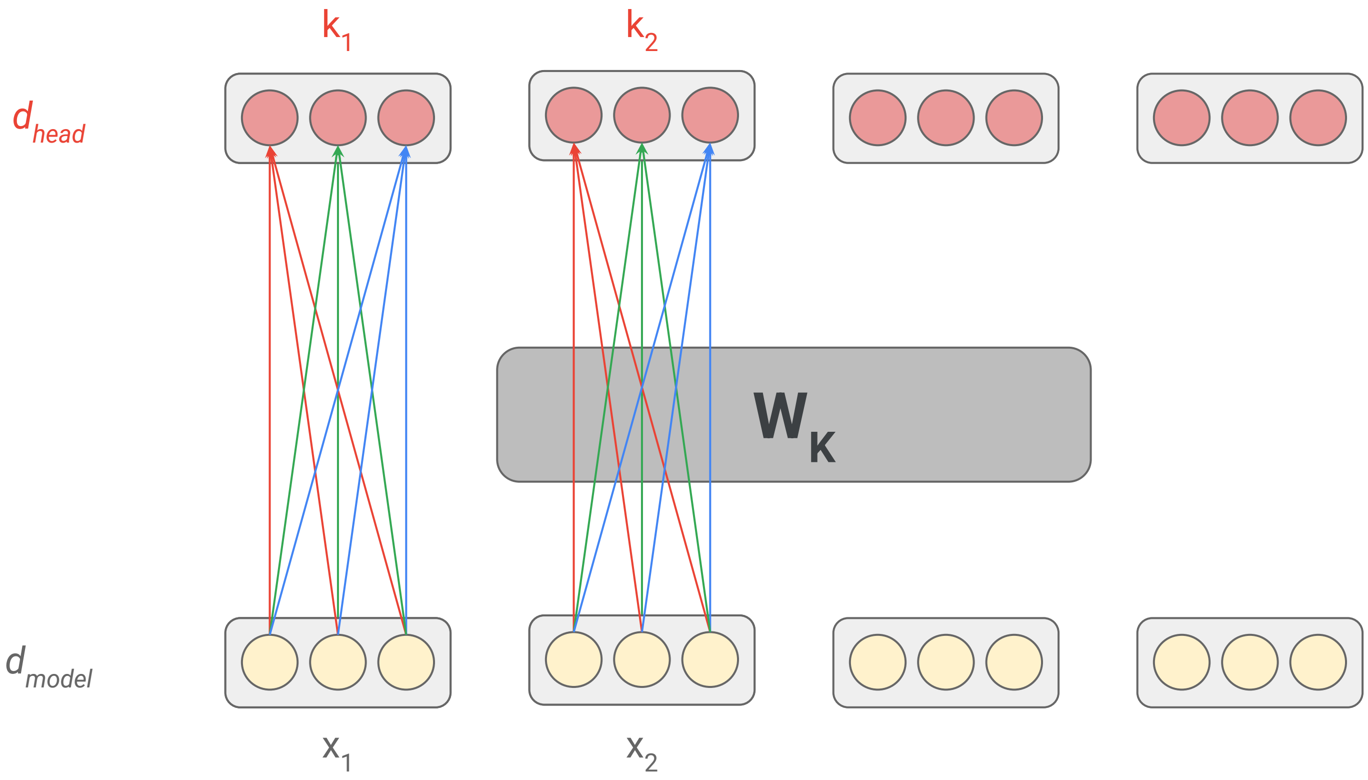
Query vectors



$$Q = X \cdot W_Q$$

linear projections

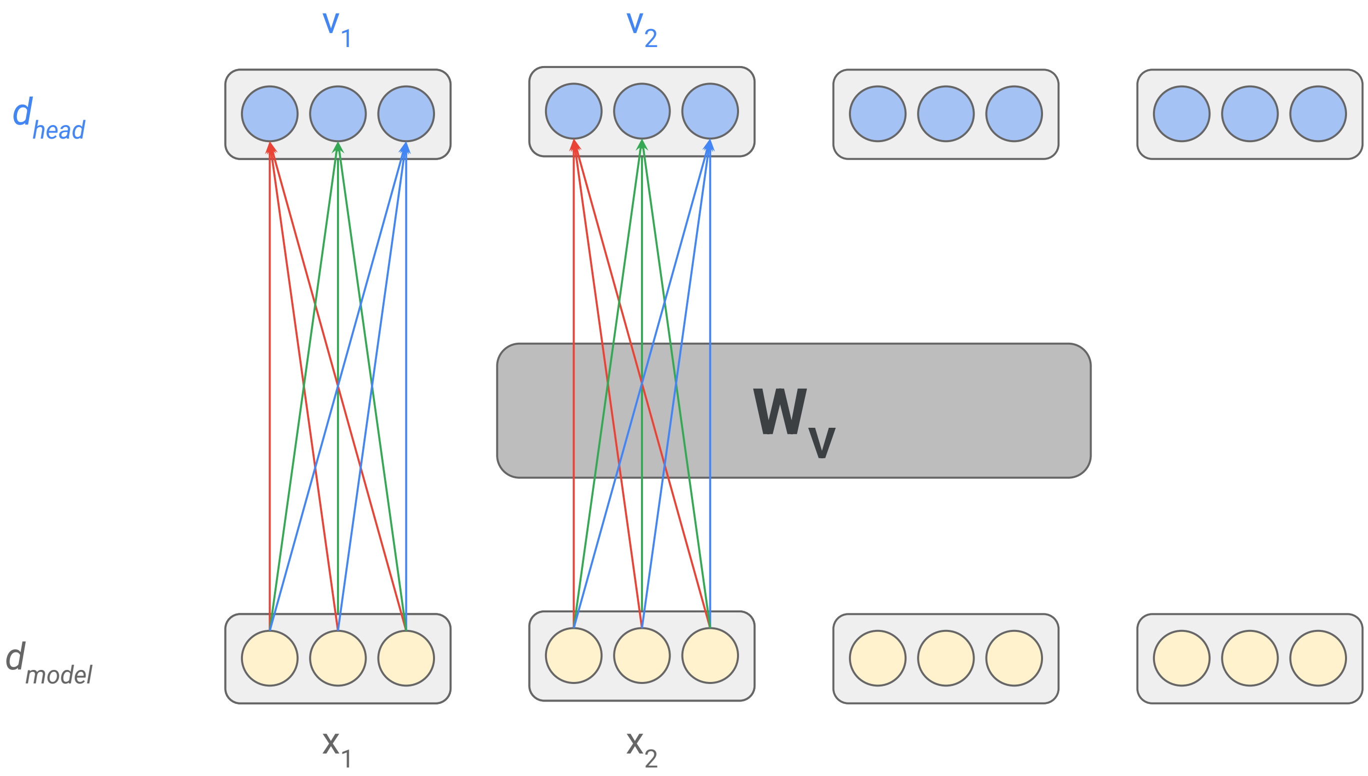
Key vectors



$K = X \cdot W_K$

linear projections

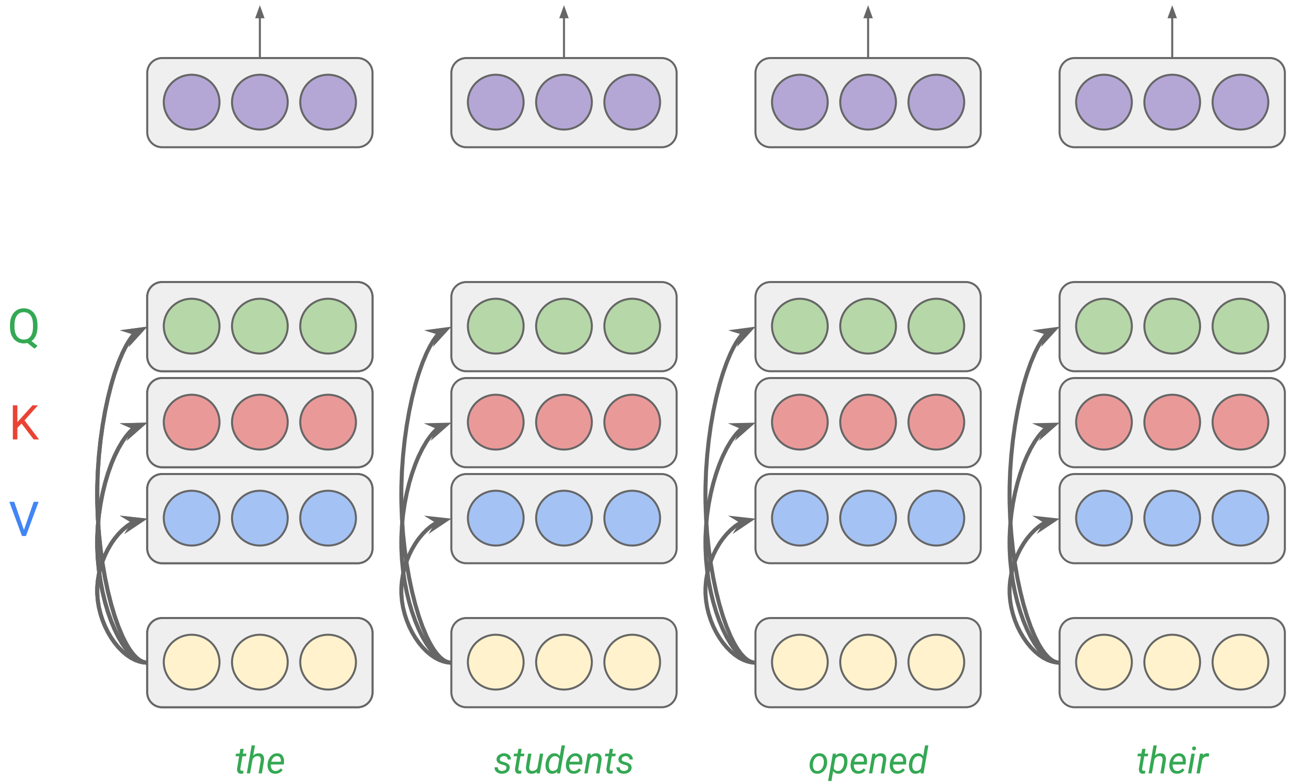
Value vectors



$$V = X \cdot W_V$$

linear projections

Attention (cont'd)



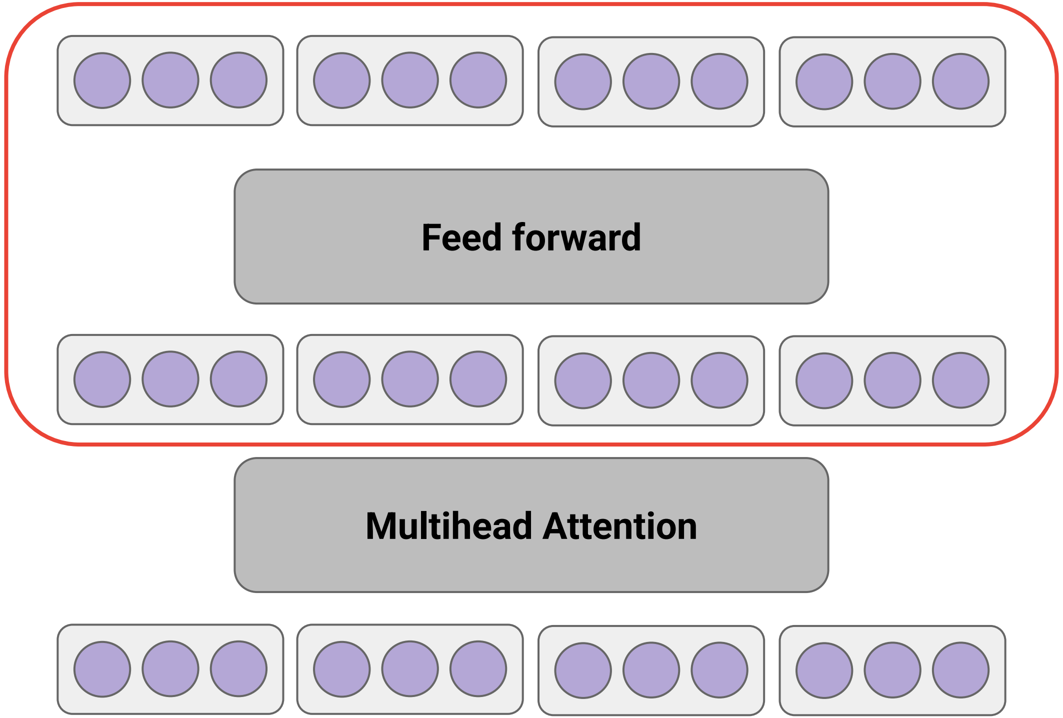
$$Q = X \cdot W_Q$$

$$K = X \cdot W_K$$

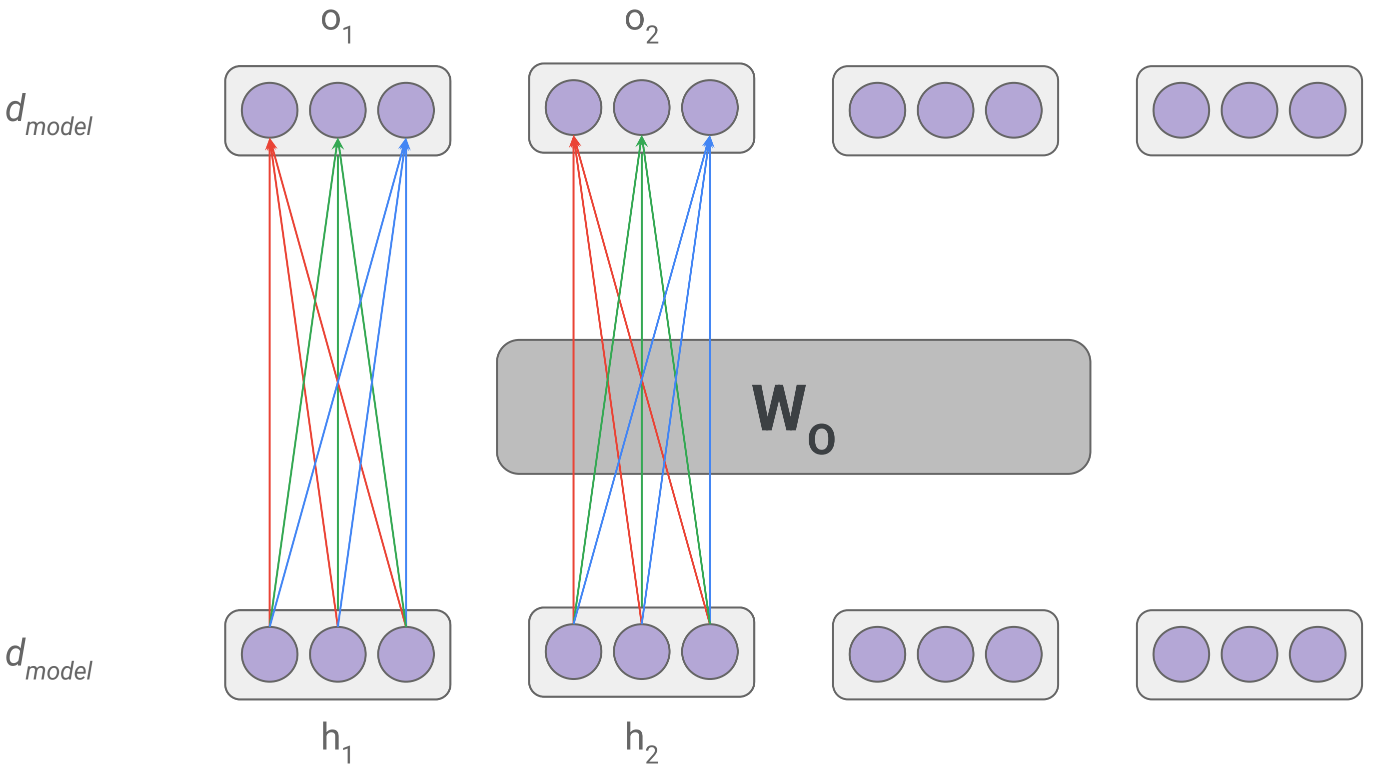
$$V = X \cdot W_V$$

linear projections

Transformer decoder



output vectors



$$O = H \cdot W_O$$

linear projections

FlashAttention: Fast and Memory-Efficient Exact Attention with IO-Awareness

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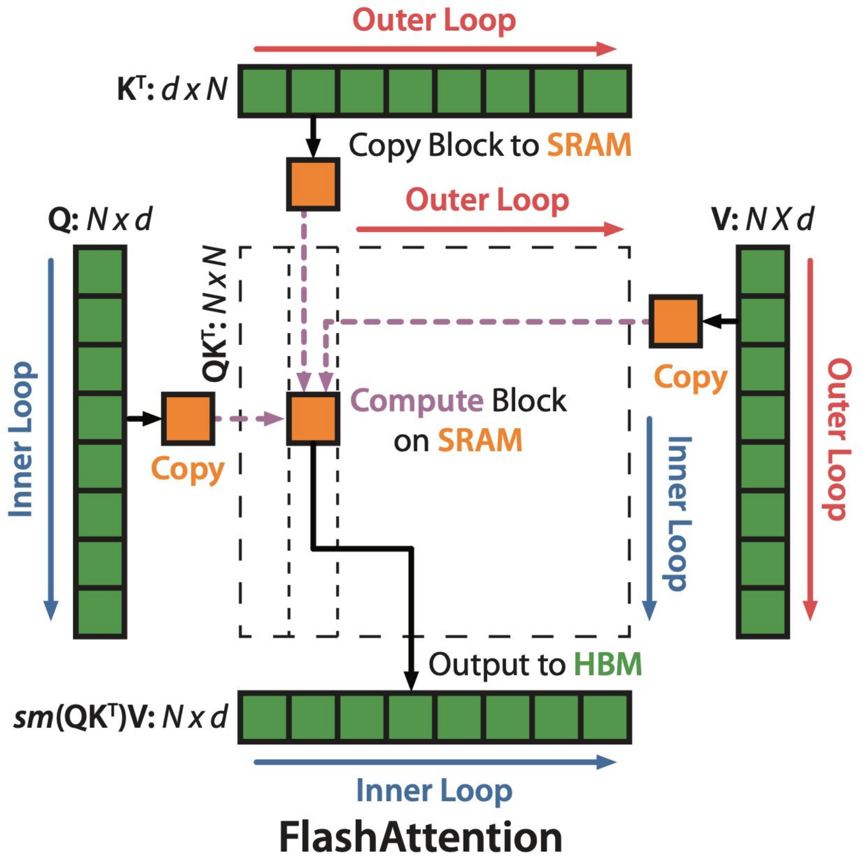
Why do we need to model longer sequences?

How to model longer sequences?

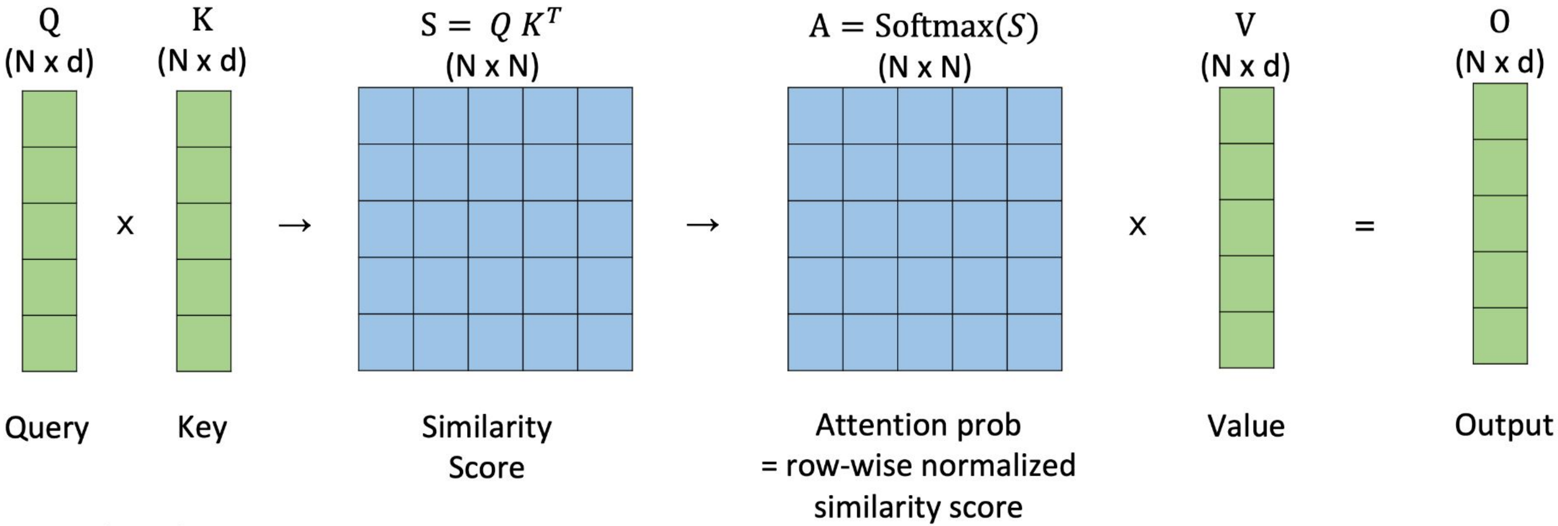
FlashAttention

Massive adoption

- **Tiling** and **recomputation** to reduce GPU memory IOs
  - **Fast** (3x) and **memory efficient** (10-20x) algorithm for **exact** attention
  - **Longer sequences** (up to 16K) yield **higher quality**



Attention mechanism review (cont'd)



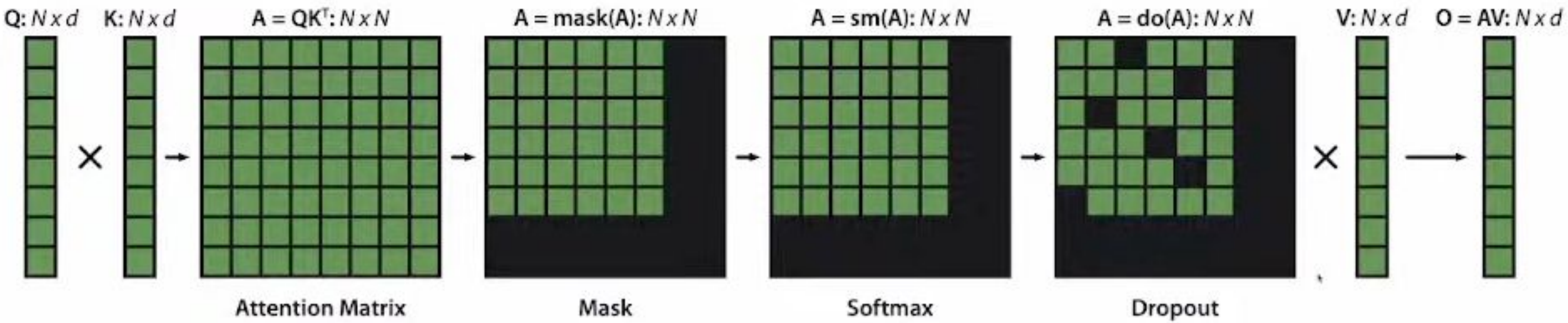
Typical sequence length N: 1K – 8K
Head dimension d: 64 – 128

$$\text{Softmax}([s_1, \cdots, s_N]) = \left[\frac{e^{s_1}}{\sum_i e^{s_i}}, \cdots, \frac{e^{s_N}}{\sum_i e^{s_i}}\right]$$

$$O = \text{Softmax}(QK^T)V$$

[FlashAttention – Tri Dao | Stanford MLSys #67]

Attention mechanism review (cont'd)



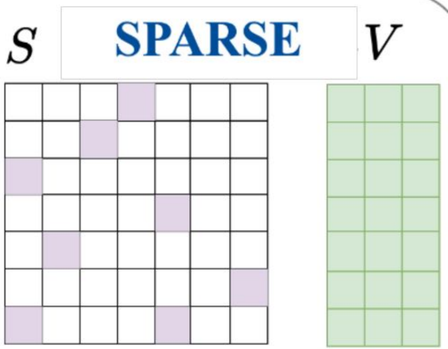
$$\mathbf{O} = \text{Dropout}(\text{Softmax}(\text{Mask}(\mathbf{QK^T})))\mathbf{V}$$

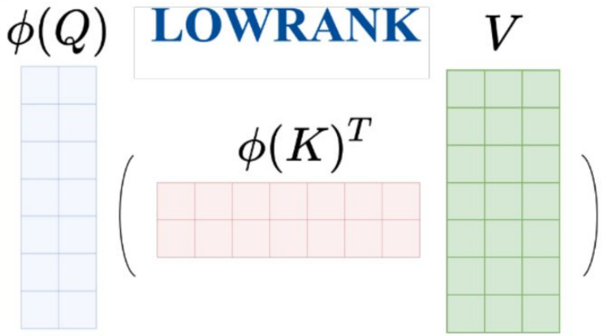
Approximate attention

tradeoff quality for ~~speed~~ fewer FLOPs

does not result in an actual wall clock speedup

Sparse Transformer
(Child et al. 19)
Reformer
(Kitaev et al. 20)
Routing Transformer
(Roy et al. 20)





Linformer
(Wang et al. 20)
Linear Transformer
(Katharopoulos et al. 20)
Performer
(Choromanski et al. 20)

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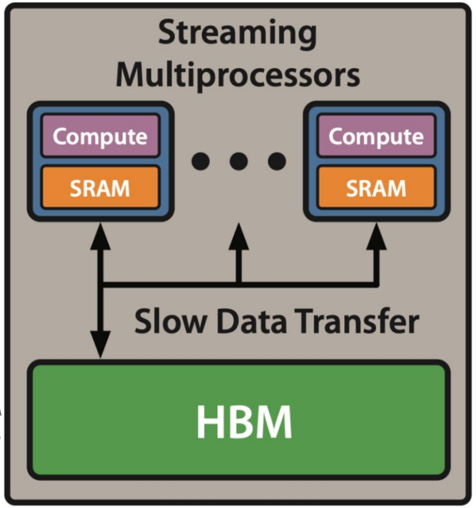
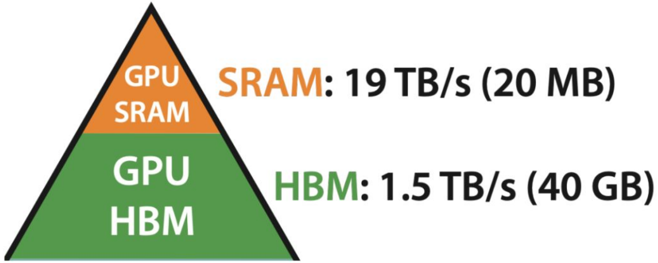
GPU compute model & memory hierarchy

2. Data moved to compute units & SRAM for computation

1. Inputs start out in HBM (GPU memory)

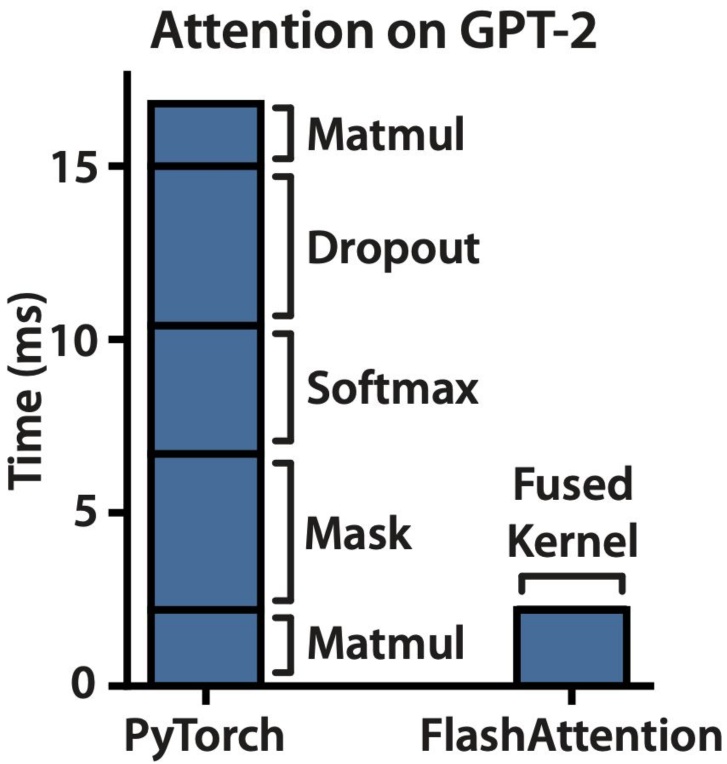
3. Output written back to HBM





Can we exploit the memory asymmetry to get speed up?

Data movement is the key bottleneck

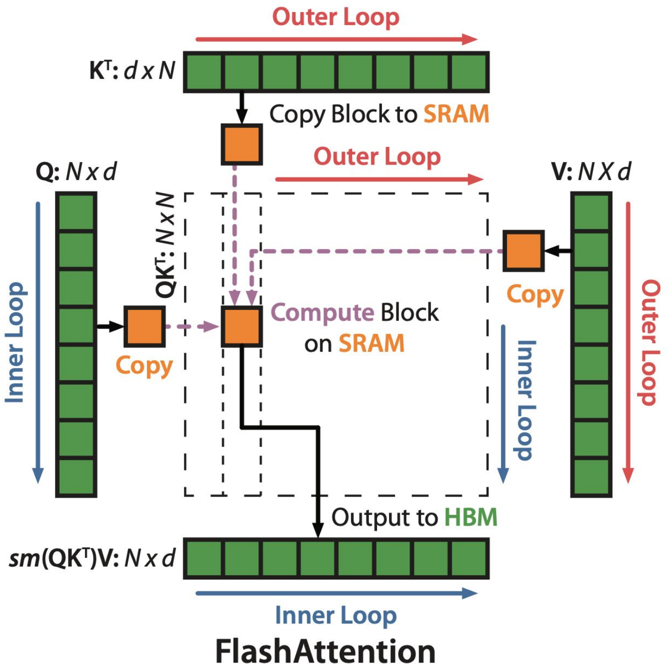


How to reduce HBM reads/writes: compute by blocks

- **Challenges:**
  - Compute softmax normalization without access to full input
  - Backward without the large attention matrix from forward
- **Approaches:**
  - **Tiling:** Restructure algorithm to load block by block from HBM to SRAM to compute attention
  - **Recomputation:** Don't store attention matrix from forward, recompute it in the backward

Tiling

- **Decomposing large softmax into smaller ones by scaling**



$$\mathrm{softmax}([A_1, A_2]) = [\alpha \times \mathrm{softmax}(A_1), \beta \times \mathrm{softmax}(A_2)]$$

$$\mathrm{softmax}([A_1, A_2]) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \alpha \times \mathrm{softmax}(A_1) V_1 + \beta \times \mathrm{softmax}(A_2) V_2$$

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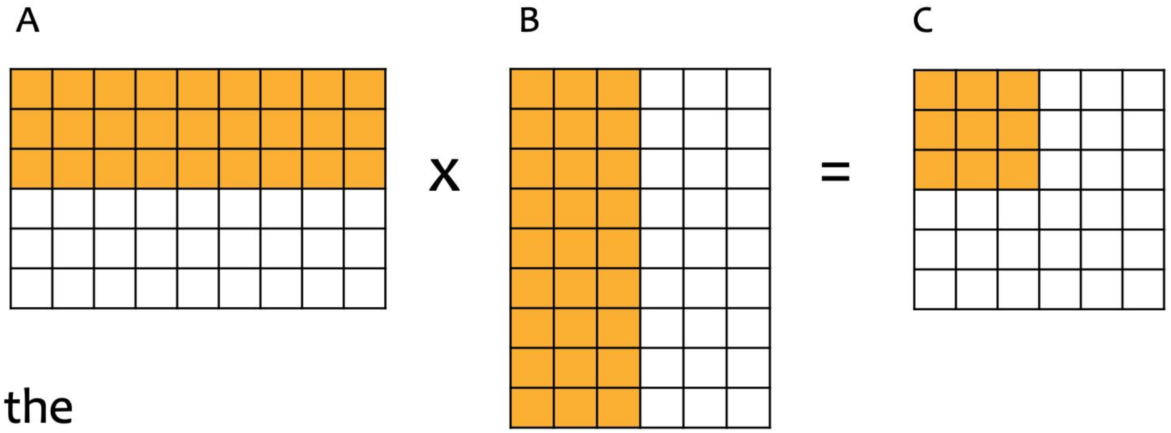
$$\mathrm{softmax}([a,b,c,d,e]) = \left[\frac{e^a}{e^a+e^b+e^c+e^d+e^e}, \frac{e^b}{e^a+e^b+e^c+e^d+e^e}, \frac{e^c}{e^a+e^b+e^c+e^d+e^e}, \frac{e^d}{e^a+e^b+e^c+e^d+e^e}, \frac{e^e}{e^a+e^b+e^c+e^d+e^e}\right]$$

$$\mathrm{softmax}([a,b,c,d,e]) = \left[\frac{e^a+e^b+e^c}{e^a+e^b+e^c+e^d+e^e} \cdot \left(\frac{e^a}{e^a+e^b+e^c}; \frac{e^b}{e^a+e^b+e^c}; \frac{e^c}{e^a+e^b+e^c}\right); \frac{e^d+e^e}{e^a+e^b+e^c+e^d+e^e} \cdot \left(\frac{e^d}{e^d+e^e}; \frac{e^e}{e^d+e^e}\right)\right]$$

; denotes concatenation
note that the terms involving $e^a + e^b + e^c$ cancel out each other
same for the $e^d + e^e$ terms

$$\mathrm{softmax}(\underbrace{[a,b,c,d,e]}_{A}) = \left[\underbrace{\frac{e^a+e^b+e^c}{e^a+e^b+e^c+e^d+e^e}}_{\alpha} \cdot \mathrm{softmax}(\underbrace{[a,b,c]}_{A_1}); \underbrace{\frac{e^d+e^e}{e^a+e^b+e^c+e^d+e^e}}_{\beta} \cdot \mathrm{softmax}(\underbrace{[d,e]}_{A_2})\right]$$

Tiling for matrix multiplication



- We can view the computation as decomposing if we consider subsets of rows/columns

$$C_{(1,1):(3,3)} = A_{(1,1):(3,9)} \times B_{(1,1):(9,3)}$$

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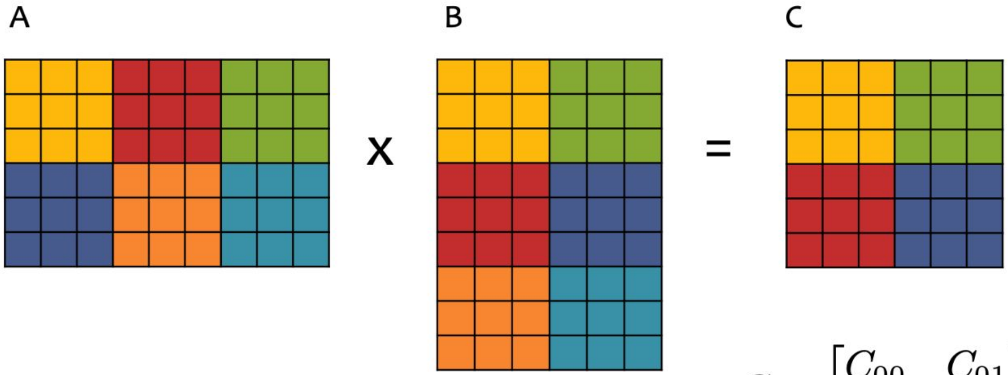
Tiling for matrix multiplication (cont'd)

- Tiling capitalizes on this decomposition
- Each output tile is computed by multiplying a pair of input tiles and adding it to the appropriate output tile



$$C = \begin{bmatrix} C_{00} & C_{01} \\ C_{10} & C_{11} \end{bmatrix}$$

with each $C_{ij} \in \mathbb{R}^{3\times3}$

$$A = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \end{bmatrix}$$

with each $A_{ij} \in \mathbb{R}^{3\times3}$

$$B = \begin{bmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \\ B_{20} & B_{21} \end{bmatrix}$$

with each $B_{ij} \in \mathbb{R}^{3\times3}$

$$C_{00} = A_{00}B_{00} + A_{01}B_{10} + A_{02}B_{20}$$
$$C_{01} = A_{00}B_{01} + A_{01}B_{11} + A_{02}B_{21}$$
$$C_{10} = A_{10}B_{00} + A_{11}B_{10} + A_{12}B_{20}$$
$$C_{11} = A_{10}B_{01} + A_{11}B_{11} + A_{12}B_{21}$$

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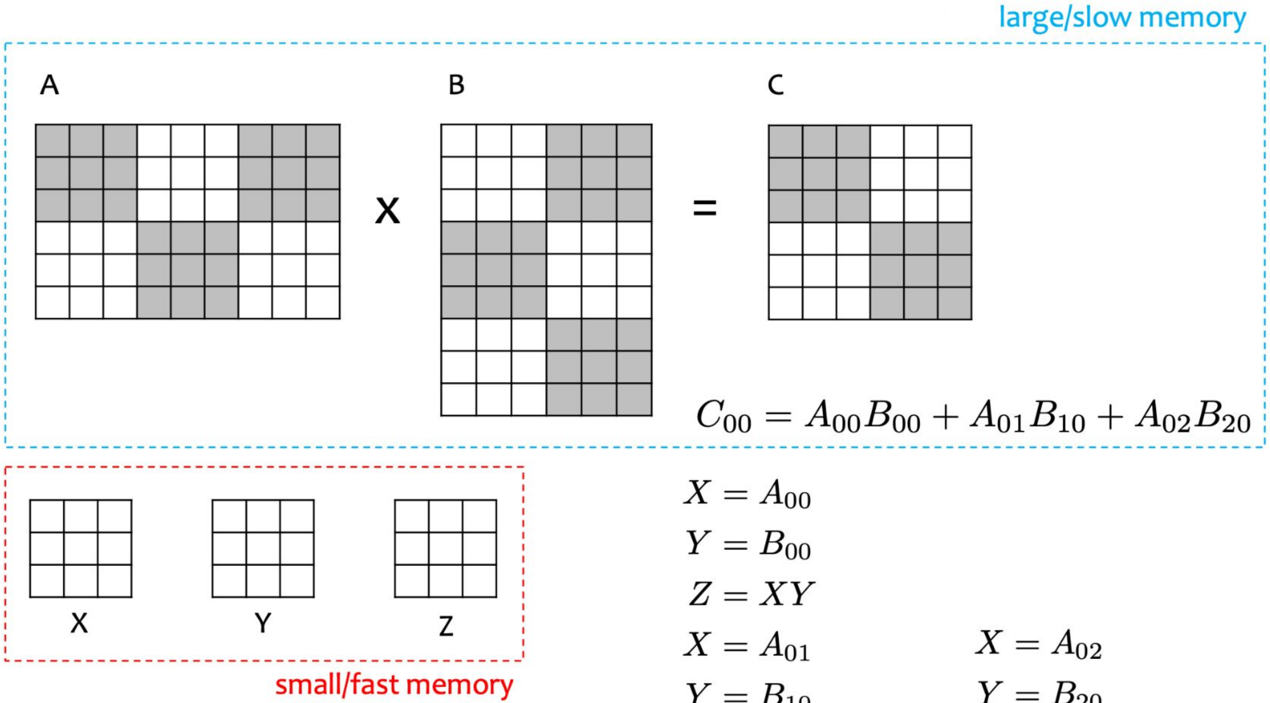
Tiling for matrix multiplication (cont'd)

- Tiling enables matrix multiplication of two **very** large matrices to capitalize on the small amount of fast memory on a device (e.g. GPU)
- Start by putting the input matrices and storage for the output matrix into large/slow memory
- Do the primary computation in slow/fast memory



$$C_{00} = A_{00}B_{00} + A_{01}B_{10} + A_{02}B_{20}$$

$$X = A_{00}$$
$$Y = B_{00}$$
$$Z = XY$$

$$X = A_{01}$$
$$Y = B_{10}$$
$$Z = Z + XY$$

$$X = A_{02}$$
$$Y = B_{20}$$
$$Z = Z + XY$$
$$C_{00} = Z$$

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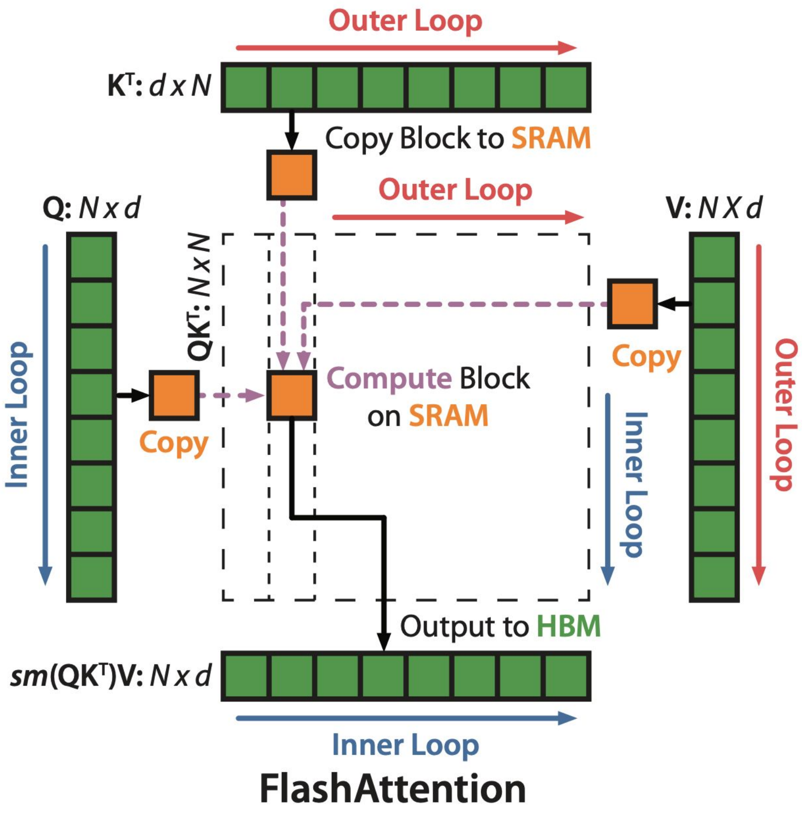
Tiling (cont'd)

1. Load inputs by blocks from HBM to SRAM.

2. On chip, compute attention output with respect to that block.

3. Update output in HBM by scaling.



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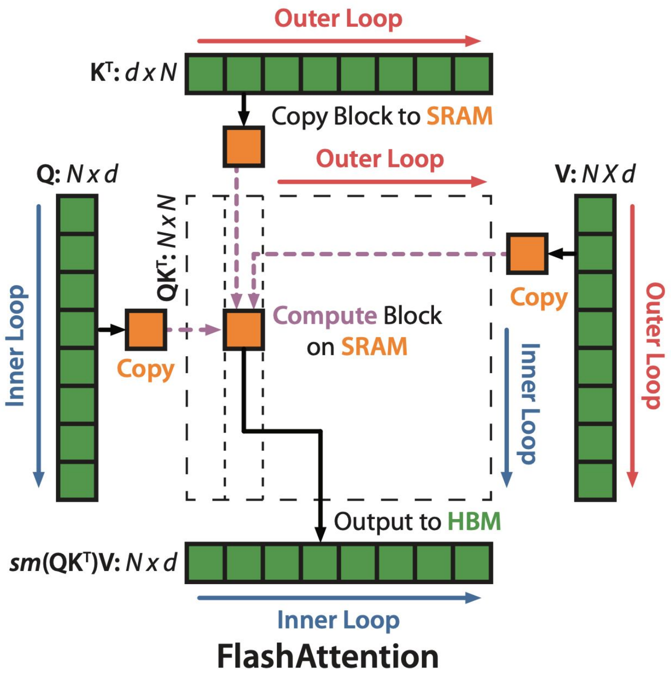
Demo

- https://jacksoncakes.com/flashattention-fast-and-memory-efficient-exact-attention/

Recomputation (backward pass)

- **By storing softmax normalization from forward (size N), quickly recompute attention in the backward from inputs in SRAM.**

Attention	Standard	FlashAttention
GFLOPs	66.6	75.2 (↑13%)
HBM reads/writes (GB)	40.3	4.4 (↓9x)
Runtime (ms)	41.7	7.3 (↓6x)



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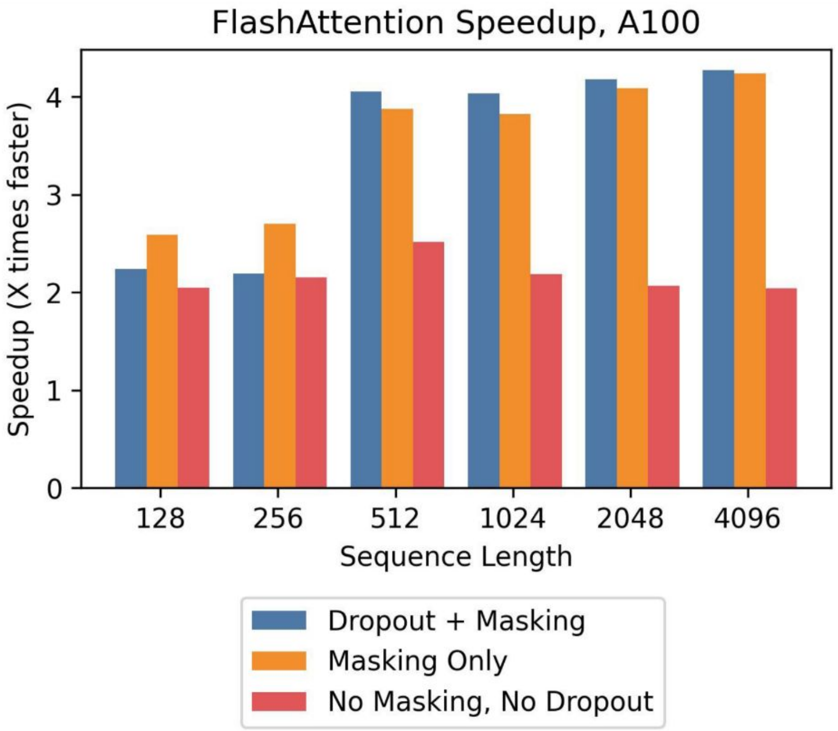
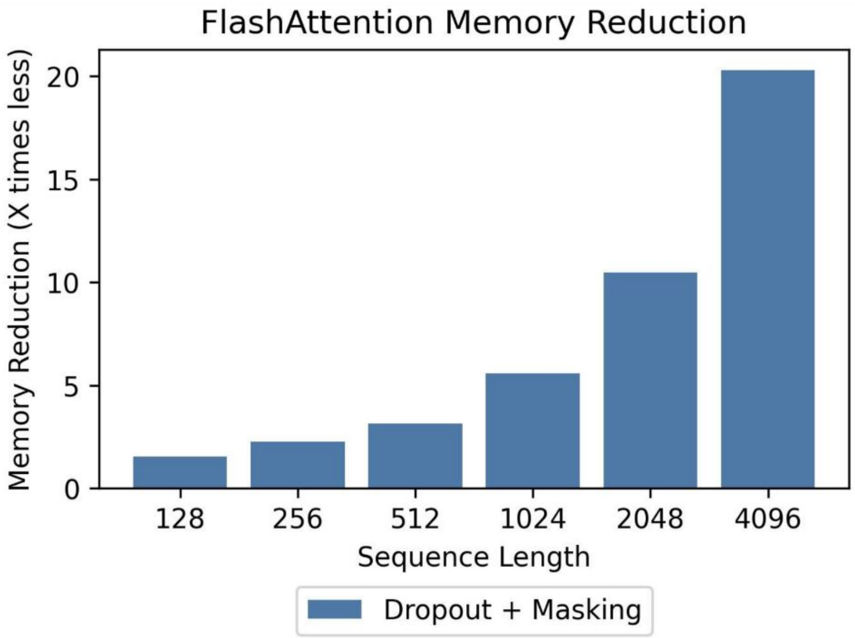
FlashAttention: 2-4x speedup, 10-20x memory reduction





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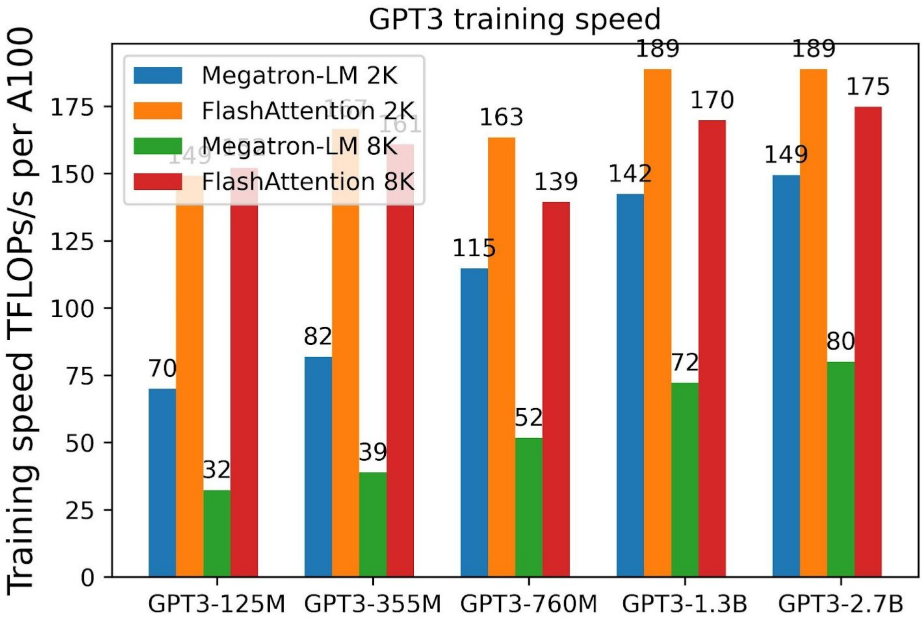
Faster Training: MLPerf Record for Training BERT-large

- MLPerf: (highly optimized) standard benchmark for training speed
- Time to hit an accuracy of 72.0% on MLM from a fixed checkpoint, averaged across 10 runs on 8 x A100 GPUs

BERT Implementation	Training time (minutes)
Nvidia MLPerf 1.1 [58]	20.0 ± 1.5
FlashAttention (ours)	$\mathbf{17.4} \pm 1.4$

Faster Training, longer context



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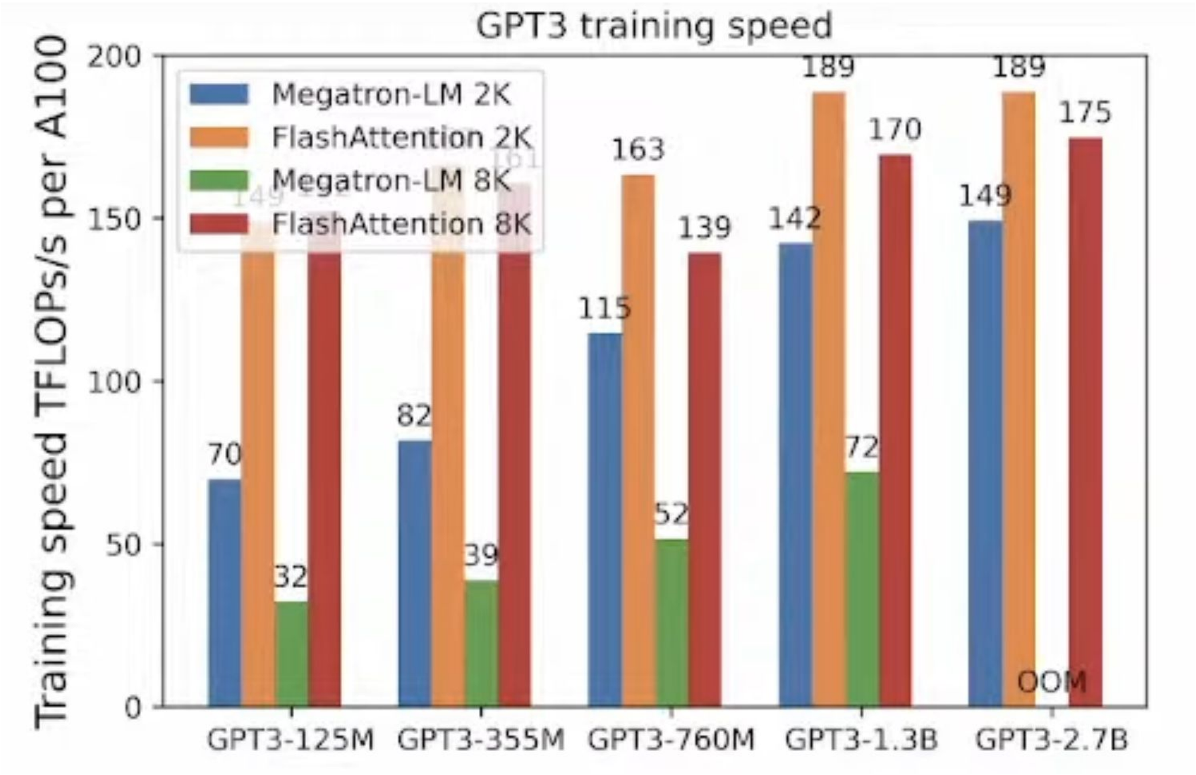
Faster Training, longer context



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Thank you!