Language modeling

CS 4804: Introduction to AI

Fall 2025

https://tuvllms.github.io/ai-fall-2025/

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Office hours (both in-person and via Zoom) will start next week. Zoom links will be posted on Piazza.

- **Contact:** Please email *all* of us at cs4804instructors@gmail.com
  For anonymous questions or comments, please use this form

Final project

- The class size has exceeded 70 students and is still growing
- Groups of **5-6**; all groups should be formed by September 5th
- A Google form for submitting group information will be available next week
- Search for teammates on Piazza https://piazza.com/class/meqiibrwtql168/post/5

  or reach out to us at cs4804instructors@gmail.com

Final project (cont'd)

*"If I were given one hour to save the planet, I would spend **59 minutes** defining the problem and one minute resolving it."*

— Albert Einstein?

Homework

- Homework 0 will be released tomorrow (due September 12th)

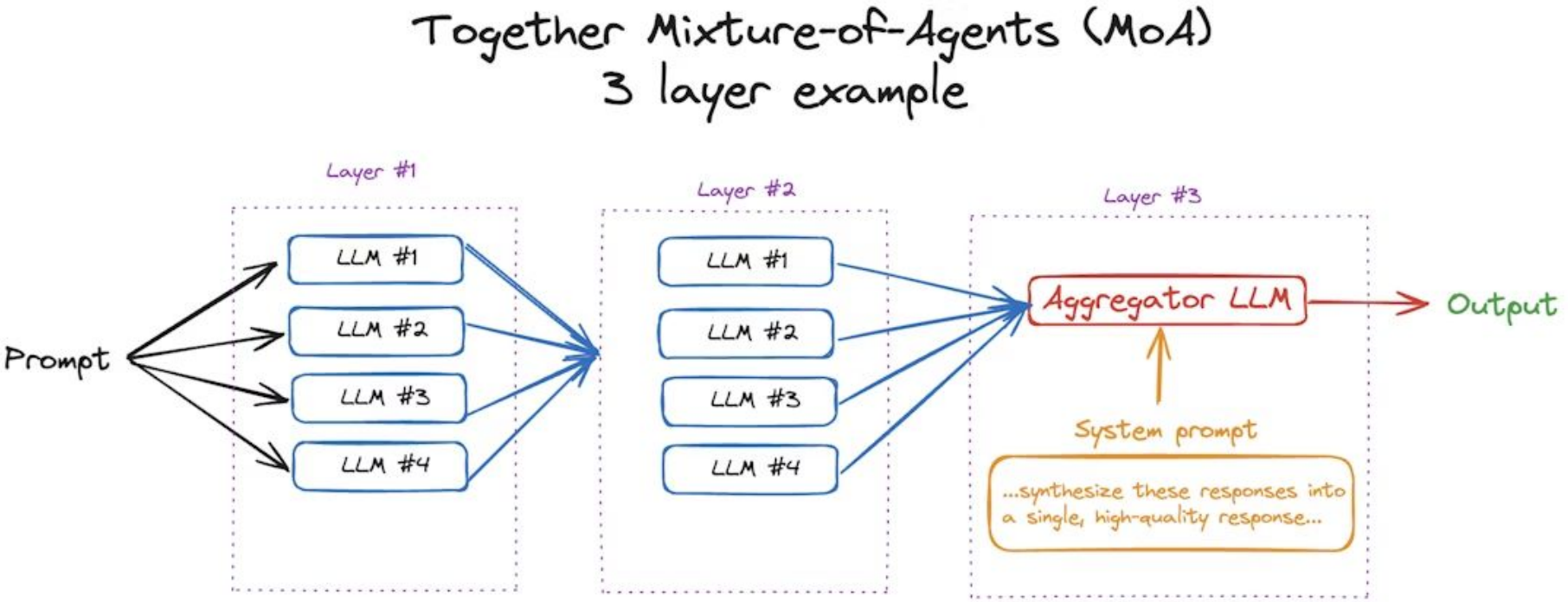
Agent framework

- **Environment:** The external world the agent operates in
- **Percepts:** The inputs the agent receives from the environment through sensors
- **State:** The agent's internal representation of the environment, derived from its percept history
- **Actions:** The outputs the agent produces through actuators that affect the environment
- **Agent function:** The abstract mapping from percepts (or percept history) to actions
- **Agent program:** The concrete implementation of the agent function in software
- **Agent architecture:** The underlying hardware or computational platform that runs the agent program

LLM agent

- **Environment:** The conversational space that includes the user, their queries, and any external systems the chatbot can access
- **Percepts:** The inputs the chatbot receives, which include user prompts, context history, and external data sources
- **State:** The chatbot's internal representation of the conversation, which is derived from the sequence of user inputs and past responses
- **Actions:** The outputs the chatbot produces, such as text completions, tool calls, or structured responses that affect the conversation or external systems
- **Agent function:** The abstract mapping from inputs (including prompt and context history) to outputs (responses or actions)
- **Agent program:** The specific software implementation of the agent function, including the trained model weights, inference code, and orchestration logic
- **Agent architecture:** The underlying computational infrastructure that runs the chatbot, including model architecture, servers, GPUs, and memory systems

Mixture-of-Agents



https://docs.together.ai/docs/mixture-of-agents#advanced-moa-example

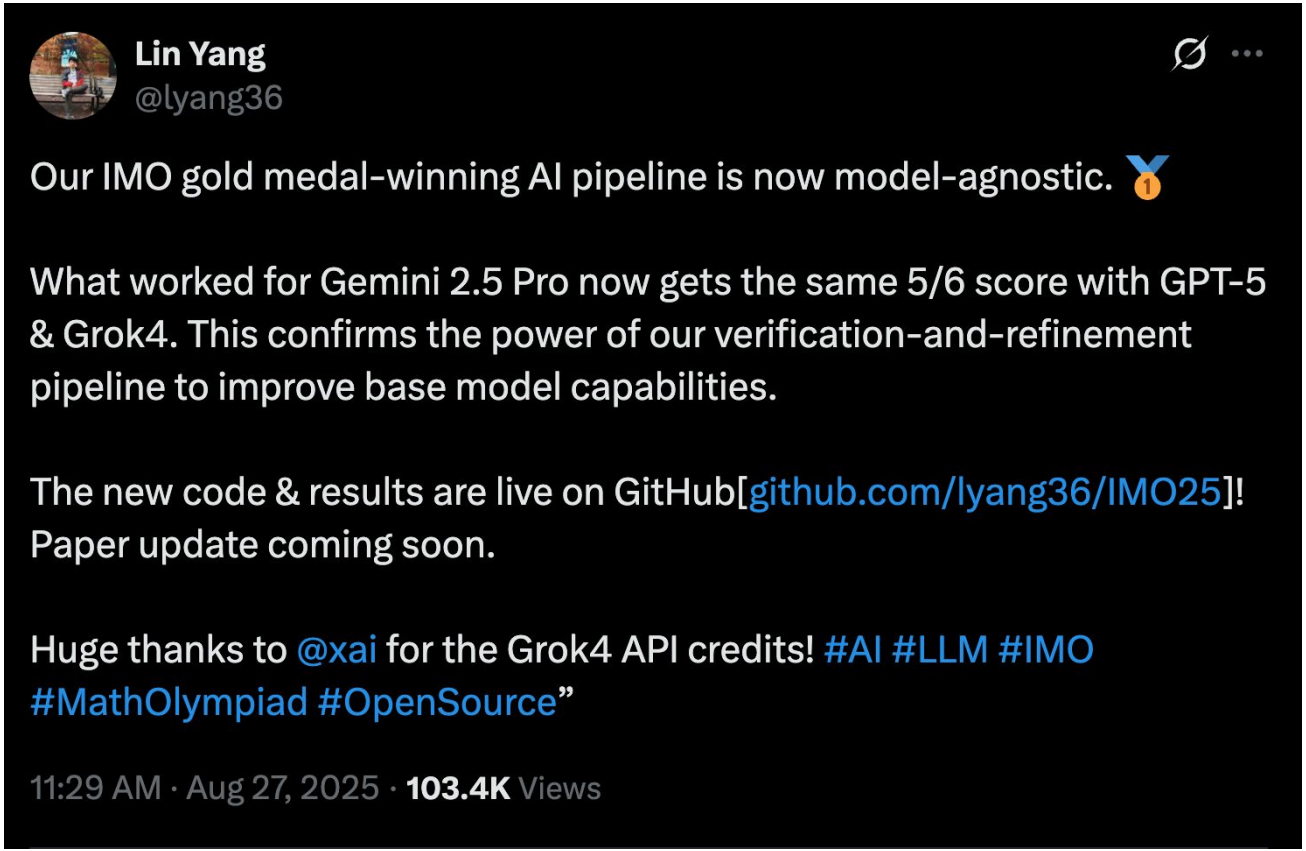
Advanced version of Gemini with Deep Think officially achieves gold-medal standard at the International Mathematical Olympiad

21 JULY 2025

Thang Luong and Edward Lockhart

Share

https://deepmind.google/discover/blog/advanced-version-of-gemini-with-deep-think-officially-achieves-gold-medal-standard-at-the-international-mathematical-olympiad/



https://x.com/lyang36/status/1960726356175806533

AgentFly: Extensible and Scalable Reinforcement Learning for LM Agents

Renxi Wang, Rifo Ahmad Genadi, Bilal El Bouardi, Yongxin Wang
Fajri Koto, Zhengzhong Liu, Timothy Baldwin, Haonan Li
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Abstract

Language model (LM) agents have gained significant attention for their ability to autonomously complete tasks through interactions with environments, tools, and APIs. LM agents are primarily built with prompt engineering or supervised finetuning. At the same time, reinforcement learning (RL) has been explored to enhance LM's capabilities, such as reasoning and factuality. However, the combination of the LM agents and reinforcement learning (Agent-RL) remains underexplored and lacks systematic study. To this end, we built AgentFly, a scalable and extensible Agent-RL framework designed to empower LM agents with a variety of RL algorithms. Our framework supports multi-turn interactions by adapting traditional RL methods with token-level masking. It features a decorator-based interface for defining tools and reward functions, enabling seamless extension and ease of use. To support high-throughput training, we implement asynchronous execution of tool calls and reward computations, and design a centralized resource management system for scalable environment coordination. We also provide a suite of prebuilt tools and environments, demonstrating the framework's effectiveness through successful agent training across multiple tasks. [1]

https://arxiv.org/abs/2507.14897

Memento: Fine-tuning LLM Agents without Fine-tuning LLMs

Huichi Zhou*[1,2], Yihang Chen*[2], Siyuan Guo[3], Xue Yan[4], Kin Hei Lee , Zihan Wang , Ka Yiu Lee[2], Guchun Zhang[2], Kun Shao[2], Linyi Yang†[2], and Jun Wang†[1]

[1]AI Centre, UCL, [2]Huawei Noah's Ark Lab, UK, [3]Jilin University, [4]Institute of Automation, CAS

Abstract

In this paper, we introduce a novel learning paradigm for Adaptive Large Language Model (LLM) agents that eliminates the need for fine-tuning the underlying LLMs. Existing approaches are often either rigid, relying on static, handcrafted reflection workflows, or computationally intensive, requiring gradient updates of LLM model parameters. In contrast, our method enables low-cost continual adaptation via memory-based online reinforcement learning. We formalise this as a Memory-augmented Markov Decision Process (M-MDP), equipped with a neural case-selection policy to guide action decisions. Past experiences are stored in an episodic memory, either differentiable or non-parametric. The policy is continually updated based on environmental feedback through a memory rewriting mechanism, whereas policy improvement is achieved through efficient memory reading (retrieval). We instantiate our agent model in the deep research setting, namely *Memento*, which attains top-1 on GAIA validation (87.88% Pass@3) and 79.40% on the test set. It reaches 66.6% F1 and 80.4% PM on the DeepResearcher dataset, outperforming the state-of-the-art training-based method, while case-based memory adds 4.7% to 9.6% absolute points on out-of-distribution tasks. Our approach offers a scalable and efficient pathway for developing generalist LLM agents capable of continuous, real-time learning without gradient updates, advancing machine learning towards open-ended skill acquisition and deep research scenarios. The code is available at `https://github.com/Agent-on-the-Fly/Memento`.

https://arxiv.org/abs/2508.16153

DEEP THINK WITH CONFIDENCE

Yichao Fu[2†]*, **Xuewei Wang**[1], **Yuandong Tian**[1], **Jiawei Zhao**[1†]
[1]Meta AI, [2]UCSD
[†]Equal contribution

Project Page: `jiaweizzhao.github.io/deepconf`

ABSTRACT

Large Language Models (LLMs) have shown great potential in reasoning tasks through test-time scaling methods like self-consistency with majority voting. However, this approach often leads to diminishing returns in accuracy and high computational overhead. To address these challenges, we introduce **Deep Think with Confidence (DeepConf)**, a simple yet powerful method that enhances both reasoning efficiency and performance at test time. DeepConf leverages model-internal confidence signals to dynamically filter out low-quality reasoning traces during or after generation. It requires no additional model training or hyperparameter tuning and can be seamlessly integrated into existing serving frameworks. We evaluate DeepConf across a variety of reasoning tasks and the latest open-source models, including Qwen 3 and GPT-OSS series. Notably, on challenging benchmarks such as AIME 2025, DeepConf@512 achieves up to 99.9% accuracy and reduces generated tokens by up to 84.7% compared to full parallel thinking.

https://arxiv.org/abs/2508.16153

Probability

- Conditional probability $P(B|A) = \frac{P(A,B)}{P(A)}$

  Rewriting

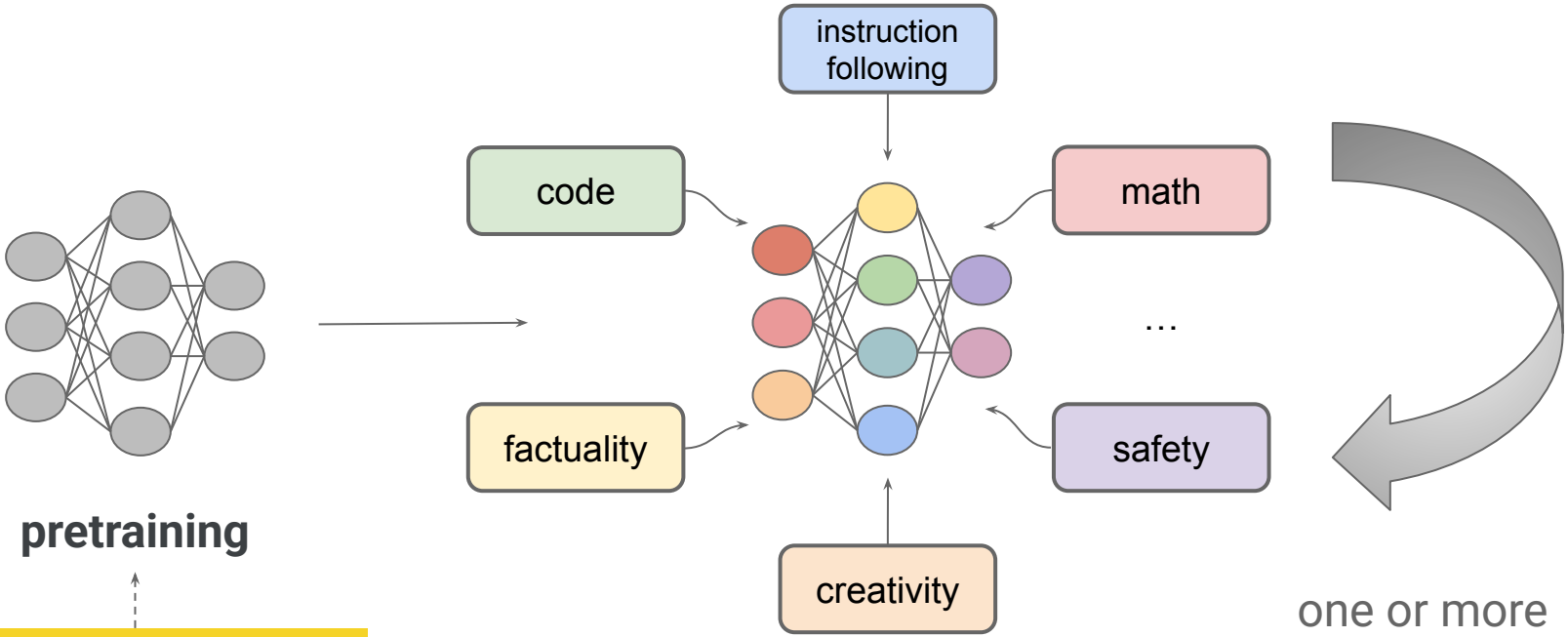
$$P(A,B) = P(A) \times P(B|A)$$

- Chain rule

$$\begin{aligned} P(X_1, X_2, \ldots, X_n) &= P(X_1, X_2, \ldots, X_{n-1}) \times P(X_n|X_1, X_2, \ldots, X_{n-1}) \\ &= P(X_1, X_2, \ldots, X_{n-2}) \times P(X_{n-1}|X_1, X_2, \ldots, X_{n-2}) \times P(X_n|X_1, X_2, \ldots, X_{n-1}) \\ &= P(X_1) \times P(X_2|X_1) \times \ldots \times P(X_n|X_1, X_2, \ldots, X_{n-1}) \end{aligned}$$

$$P(w_1, w_2, \ldots, w_n) = P(w_1) \times P(w_2|w_1) \times \ldots \times P(w_n|w_1, w_2, \ldots, w_{n-1})$$

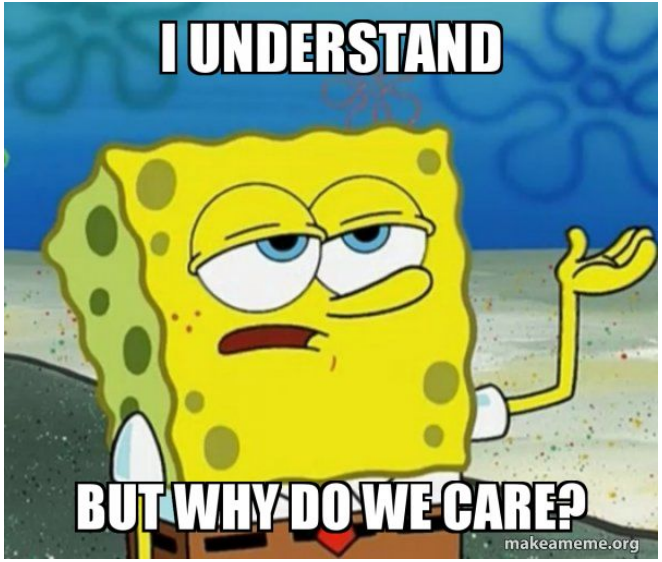
The development of modern LLMs



Language modeling

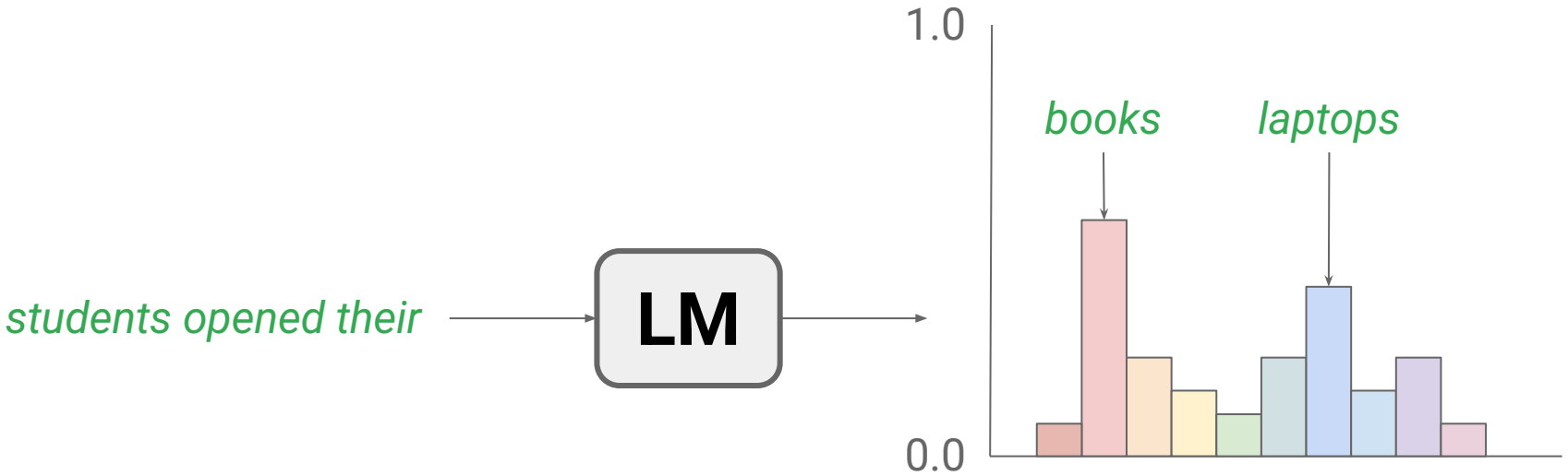
- **Predicting the next/missing word**

  Example: “The cat is on the ___.” → Predicted: “mat”.



What is a language model?

- A machine learning model that assigns a ***probability*** to each possible next word, or a ***probability distribution*** over possible next words



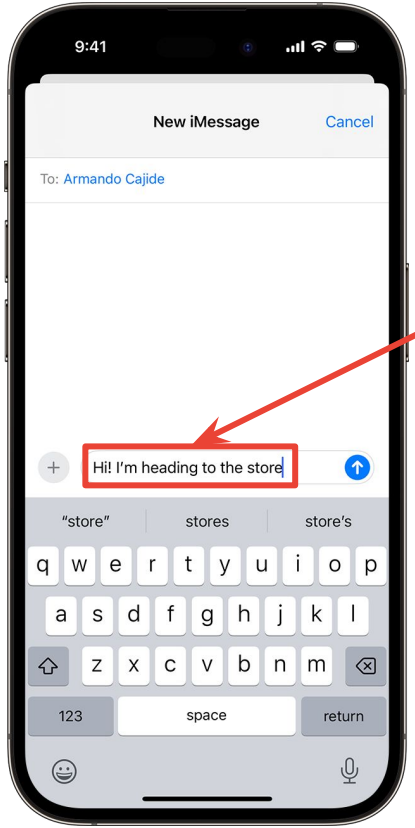
What is a language model? (cont'd)

- A language model can also assign a probability to an entire sentence

$$P(w_1, w_2, \ldots, w_n) = P(w_1) \times P(w_2|w_1) \times \ldots \times P(w_n|w_1, w_2, \ldots, w_{n-1})$$

P(*"The cat is on the mat"*) = ***P***(*"The"*) *x* ***P***(*"cat" | "The"*) *x* ***P***(*"is" | "The cat"*) *x* ***P***(*"on" | "The cat is"*) *x* ***P***(*"the" | "The cat is on"*) *x* ***P***(*"mat" | "The cat is on the"*)

You use language models everyday!



prefix

Google

san f

san francisco weather
san francisco
san francisco giants
san fernando valley
san francisco state university
san francisco hotels
san francisco 49ers
san fernando
san fernando mission
san francisco zip code

Google Search | I'm Feeling Lucky

source: Apple Support

source: Google Blog

Two categories of language models

- Statistical/Probabilistic language models
  - N-gram / Count-based language models

- Neural language models (e.g., ChatGPT, Gemini)

N-grams

- An n-gram is a sequence of n words
- Unigram (n=1)
  - “The”, “water”, “of”, “Walden”, “Pond”
- Bigram (n=2)
  - “The water”, “water of”, “of Walden”, “Pond”
- Trigram (n=3)
  - “The water of”, “water of Walden”, “of Walden Pond”
- 4-gram
- …

N-grams (cont'd)

- Notation
  - **word type**: a unique word in our vocabulary
  - **token**: an individual occurrence of a word type

Example: "I am Sam. Sam am I. I do not like green eggs and ham."

→ one word type of "I", three tokens of "I"

N-grams (cont'd)

- How to compute the probabilities?

$$P(w_1, w_2, \ldots, w_n) = P(w_1) \times P(w_2|w_1) \times \ldots \times P(w_n|w_1, w_2, \ldots, w_{n-1})$$

***P**(“blue” | “The water of Walden Pond is so beautifully”)*

What is the problem with this approach?

The Markov assumption

- n-gram model: Approximate the prefix by just the last ***n-1*** words
- bigram (n=2) model

$$P(\text{“blue”} \mid \text{“The water of Walden Pond is so beautifully”}) = P(\text{“blue”} \mid \text{beautifully”})$$

- trigram (n=3) model

$$P(\text{“blue”} \mid \text{“The water of Walden Pond is so beautifully”}) = P(\text{“blue”} \mid \text{so beautifully”})$$

The Markov assumption (cont'd)

- unigram model

$$P(w_1, w_2, \ldots, w_n) = P(w_1) \times P(w_2|w_1) \times \ldots \times P(w_n|w_1, w_2, \ldots, w_{n-1})$$
$$\approx P(w_1) \times P(w_2) \times \cdots \times P(w_n)$$
$$= \prod_{k=1}^{n} P(w_k)$$

- bigram model

$$P(w_1, w_2, \ldots, w_n) \approx P(w_1) \times P(w_2|w_1) \times \cdots \times P(w_n|w_{n-1})$$
$$= \prod_{k=1}^{n} P(w_k|w_{k-1})$$

Maximum likelihood estimation (MLE)

$$P(w_n|w_{n-1}) = \frac{Count(w_{n-1}w_n)}{\sum_w Count(w_{n-1}w)} = \frac{Count(w_{n-1}w_n)}{Count(w_{n-1})}$$

relative frequency

<s> I am Sam </s>
<s> Sam I am </s>
<s> I do not like green eggs and ham </s>

Here are the calculations for some of the bigram probabilities

$P(\text{I}|\text{<s>}) = \frac{2}{3} = 0.67$ $P(\text{Sam}|\text{<s>}) = \frac{1}{3} = 0.33$ $P(\text{am}|\text{I}) = \frac{2}{3} = 0.67$

$P(\text{</s>}|\text{Sam}) = \frac{1}{2} = 0.5$ $P(\text{Sam}|\text{am}) = \frac{1}{2} = 0.5$ $P(\text{do}|\text{I}) = \frac{1}{3} = 0.33$

source: Jurafsky and Martin

Example

- From a restaurant corpus

  “can you tell me about any good cantonese restaurants close by”

  “tell me about chez panisse”

  “i’m looking for a good place to eat breakfast”

  “when is caffe venezia open during the day”

source: Jurafsky and Martin

Example (cont'd)

unigram counts

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

target

prefix

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

want followed **i** 827 times

source: Jurafsky and Martin

Example (cont'd)

827/2533

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Here are a few other useful probabilities:

$P(\texttt{i}|\texttt{<s>}) = 0.25$

$P(\texttt{food}|\texttt{english}) = 0.5$

$P(\texttt{english}|\texttt{want}) = 0.0011$

$P(\texttt{</s>}|\texttt{food}) = 0.68$

$$
\begin{aligned}
P(\texttt{<s> i want english food </s>}) &= P(\texttt{i}|\texttt{<s>})P(\texttt{want}|\texttt{i})P(\texttt{english}|\texttt{want}) \\
&\quad P(\texttt{food}|\texttt{english})P(\texttt{</s>}|\texttt{food}) \\
&= 0.25 \times 0.33 \times 0.0011 \times 0.5 \times 0.68 \\
&= 0.000031
\end{aligned}
$$

source: Jurafsky and Martin

Example (cont'd)

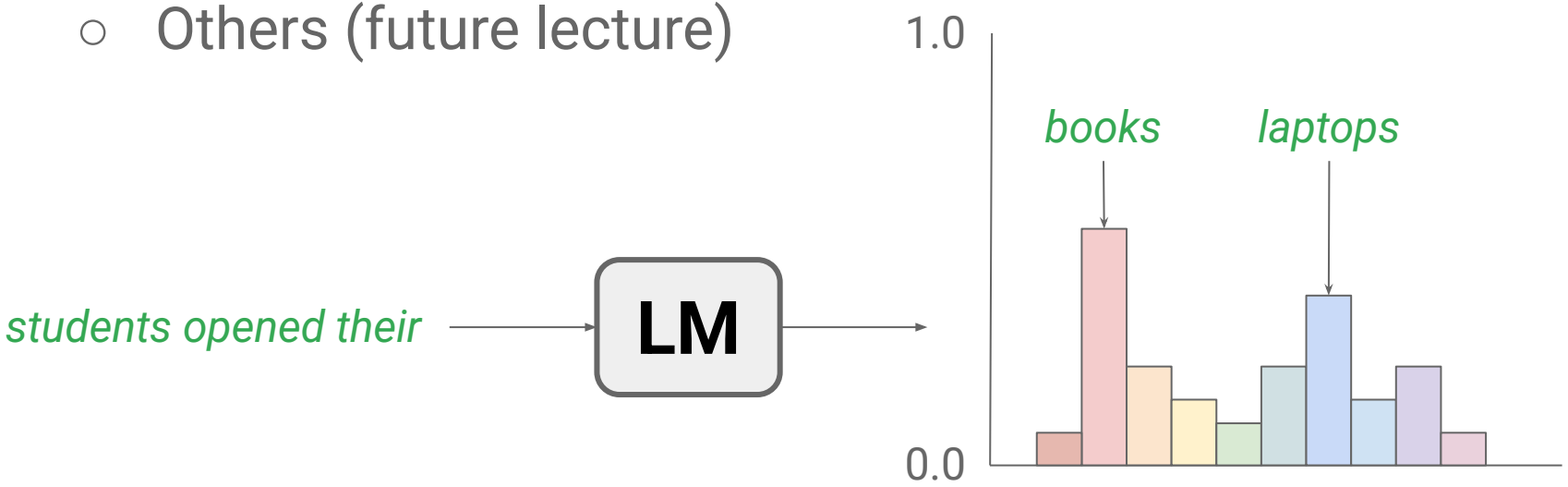
sparsity issue

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

source: Jurafsky and Martin

How to sample sentences from a language model?

- Decoding strategies
  - Greedy decoding
  - Sampling
  - Others (future lecture)



Sample generations

1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have
	–Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.
	–What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.
	–This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;
	–It cannot be but so.

from King John

Figure 3.4 Eight sentences randomly generated from four n-grams computed from Shakespeare's works. All characters were mapped to lower-case and punctuation marks were treated as words. Output is hand-corrected for capitalization to improve readability.

source: Jurafsky and Martin

Is a 4-gram model sufficient for language modeling?

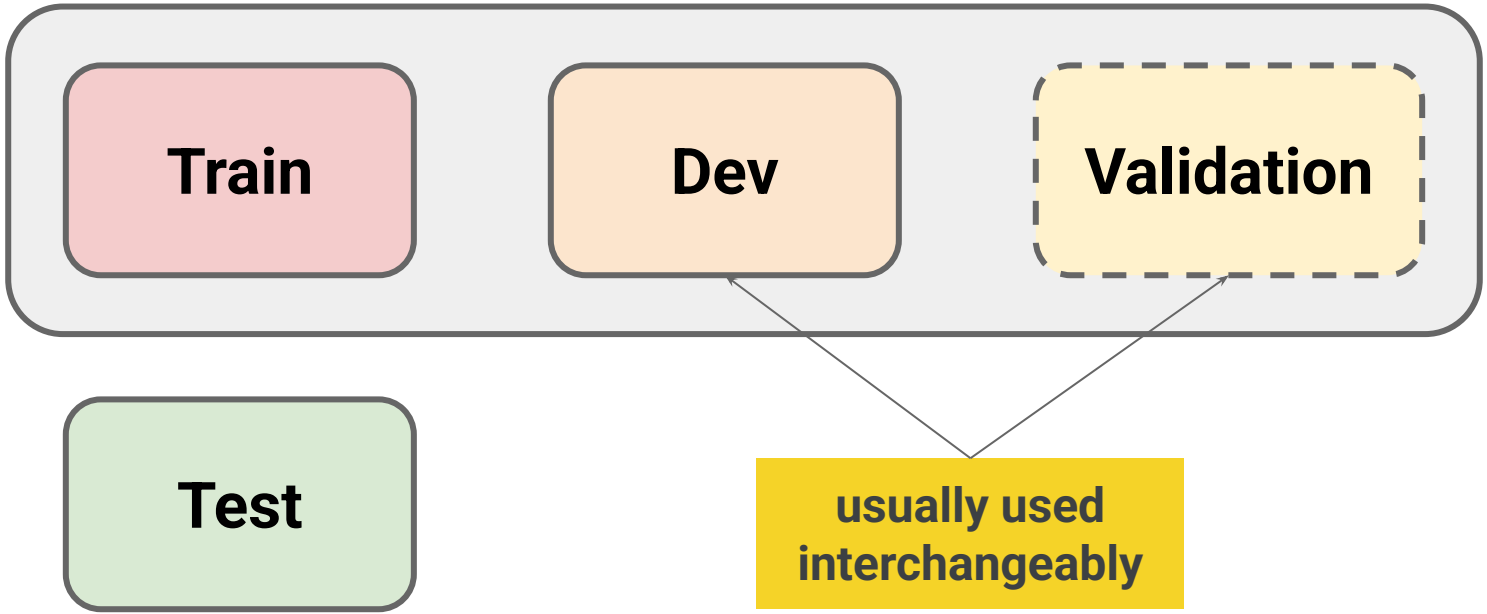
Should we increase the value of n?

- As n increases, the number of possible n-grams grows exponentially (many n-grams have insufficient or no data)
- Storing and processing large n-grams requires more memory and computational power
- Beyond a certain point, increasing n may not yield significant performance improvements, especially if the dataset does not contain sufficient examples of longer n-grams

Shakespeare as corpus

- T=884,647 tokens, V=29,066
- Shakespeare produced 300,000 bigram types out of V^2= 844,000,000 possible bigrams.
- **99.96%** of the possible bigrams have zero entries in the bigram table (were never seen)!

Evaluating language models



Never train on the test set!

Susan Zhang ✓
@suchenzang

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MBPP might've also been used somewhere in the Phi-1.5 dataset.

Just like we truncated one of the GSM8K problems, let's try truncating the MBPP prompts to see what Phi-1.5 will autocomplete with.

[h/t to @drjwrae for suggesting this too: x.com/drjwrae/status...]

Part 2

> **Susan Zhang** ✓ @suchenzang · Sep 12, 2023
> I think Phi-1.5 trained on the benchmarks. Particularly, GSM8K.
>
> x.com/suchenzang/sta...

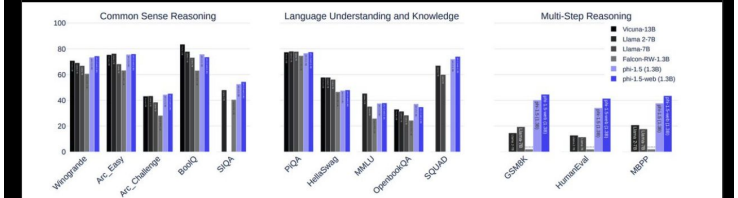

Figure 1: Benchmark results comparing **phi-1.5**, its version enhanced with filtered web data **phi-1.5-web**, and other state-of-the-art open-source LLMs. Sizes range from **phi-1.5**'s 1.3 billion parameters (Falcon-RW-1.3B [PMH+23]) to 10x larger models like Vicuna-13B [ZCS+23], a fine-tuned version of Llama-13B [TLI+23]). Benchmarks are broadly classified into three categories: common sense reasoning, language skills, and multi-step reasoning. The classification is meant to be taken loosely, for example while HellaSwag requires common sense reasoning, it arguably relies more on "memorized knowledge". One can see that **phi-1.5** models perform comparable in common sense reasoning and language skills, and vastly exceeds other models in multi-step reasoning. Note that the numbers are from our own evaluation pipeline, to ensure consistency between models, and thus they might differ slightly from numbers reported elsewhere.

Susan Zhang ✓
@suchenzang

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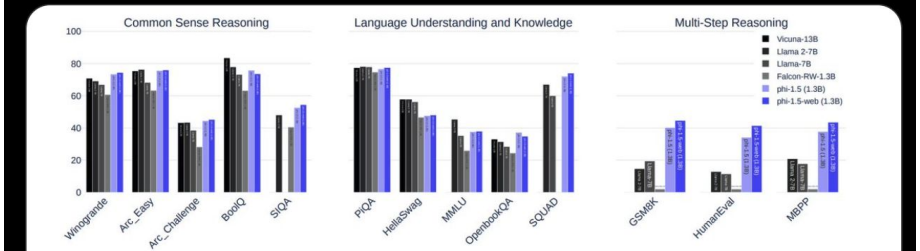

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> **Susan Zhang** ✓ @suchenzang · Aug 2, 2023
> Never trust a result in 2023 that doesn't mention the risk of dataset contamination. x.com/mathemagic1an/...

Perplexity

$$\text{perplexity}(W) = P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}}$$

We normalize by the number of words N by taking the Nth root

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}}$$

Or we can use the chain rule to expand the probability of W:

$$\text{perplexity}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i | w_1 \ldots w_{i-1})}}$$

Perplexity as Weighted Average Branching Factor

- Suppose a sentence consists of random digits.
  What is the perplexity of this sentence for a model that assigns a probability of 1/10 to each digit?

$$\text{PP}(W) = P(w_1 w_2 \dots w_N)^{-\frac{1}{N}}$$

$$= \left(\frac{1}{10^N}\right)^{-\frac{1}{N}}$$

$$= \frac{1}{10}^{-1}$$

$$= 10$$

Lower perplexity = Better language model

	Unigram	Bigram	Trigram
Perplexity	962	170	109

source: Jurafsky and Martin

In practice, we use log probs

$$log \prod p(w_i|w_{i-1}) = \sum logp(w_i|w_{i-1})$$

logs to avoid numerical underflow

sentence: I love love love love love the movie

$$p(\text{i}) \cdot p(\text{love})^5 \cdot p(\text{the}) \cdot p(\text{movie}) = 5.95374181\text{e-}7$$

$$\log p(\text{i}) + 5 \log p(\text{love}) + \log p(\text{the}) + \log p(\text{movie})$$

$$= -14.3340757538$$

source: Mohit Iyyer

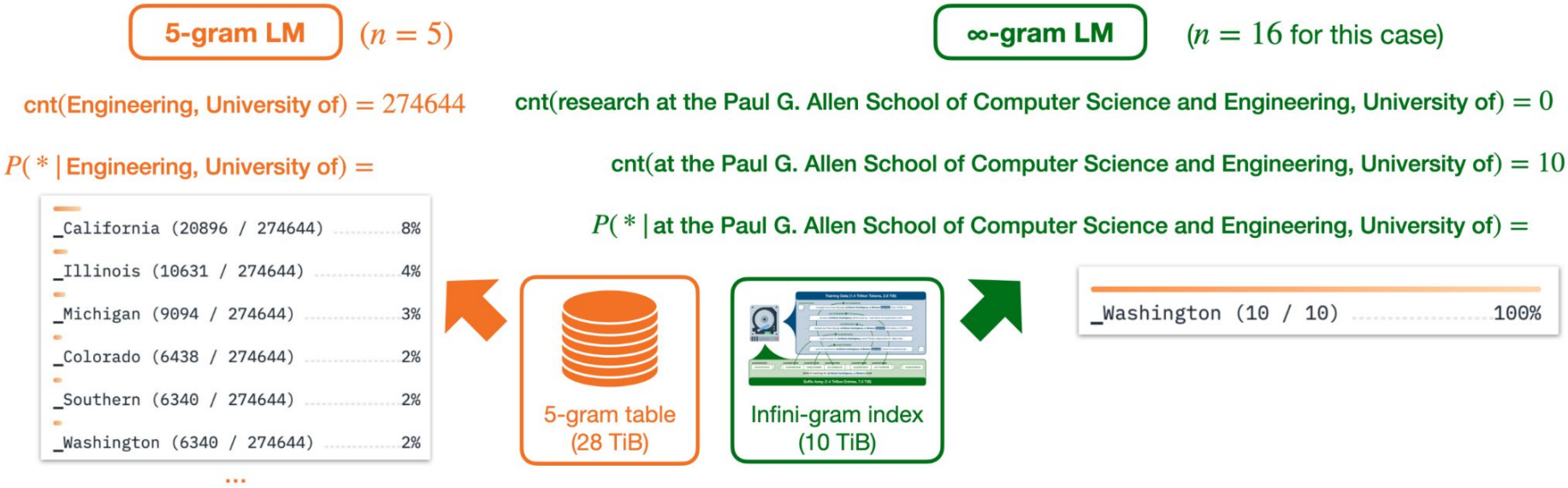
In practice, we use log probs (cont'd)

$$perplexity(W) = exp(-\frac{1}{N}\sum_{i}^{N} logp(w_i|w_{<i}))$$

perplexity is the exponentiated token-level negative log-likelihood

source: Mohit Iyyer

Infini-gram: Scaling Unbounded n-gram Language Models to a Trillion Tokens



https://arxiv.org/abs/2401.17377

Thank you!