Long-context LLMs

CS 5624: Natural Language Processing

Spring 2025

https://tuvllms.github.io/nlp-spring-2025

Tu Vu

VIRGINIA TECH

FlashAttention: Fast and Memory-Efficient Exact Attention with IO-Awareness

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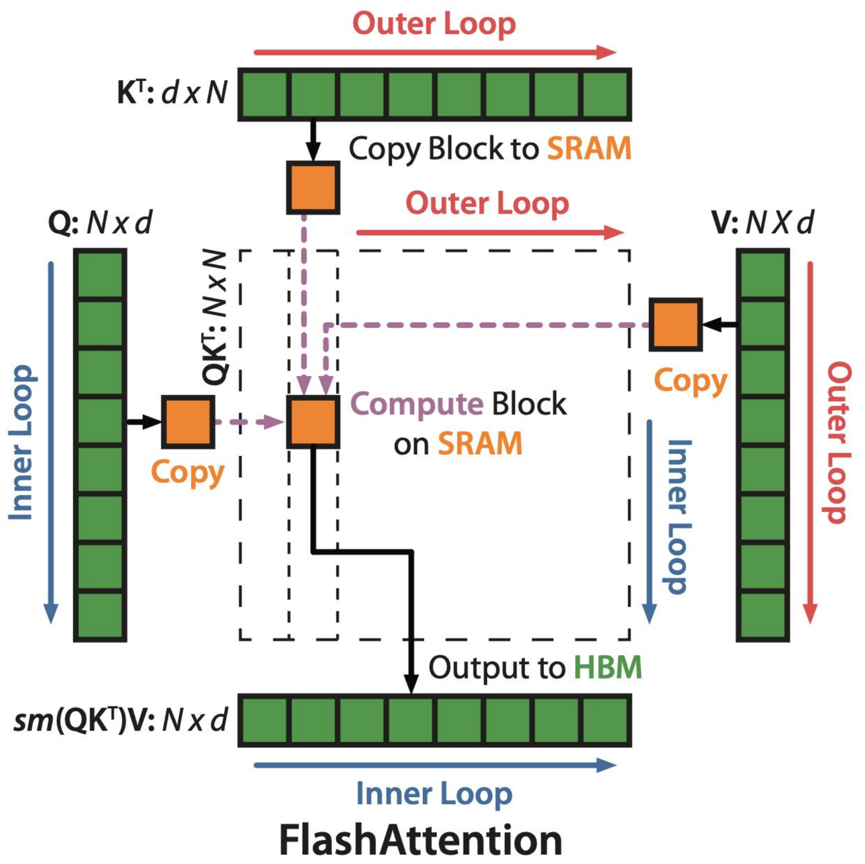
Why do we need to model longer sequences?

How to model longer sequences?

FlashAttention

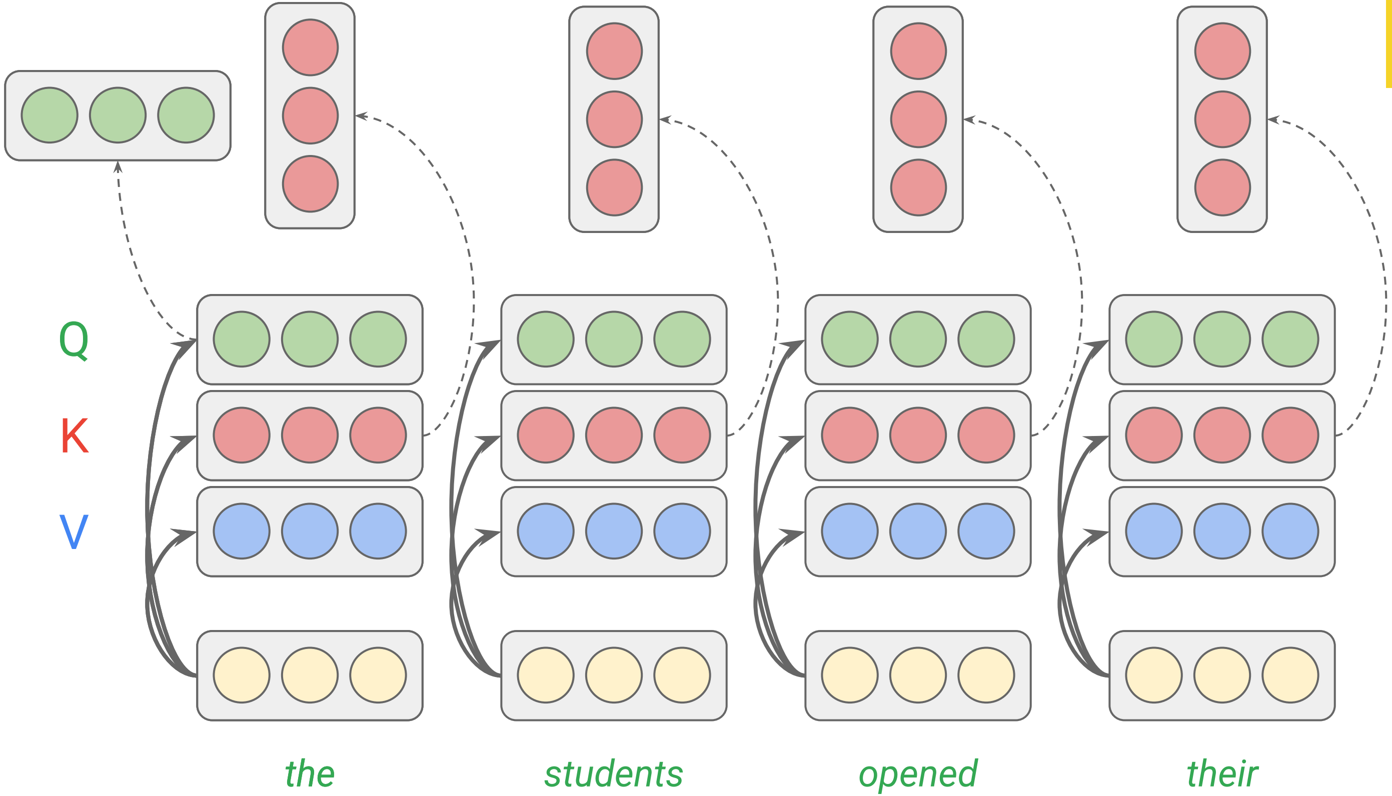
Massive adoption

- **Tiling** and **recomputation** to reduce GPU memory IOs
  - **Fast** (3x) and **memory efficient** (10-20x) algorithm for **exact** attention
  - **Longer sequences** (up to 16K) yield **higher quality**



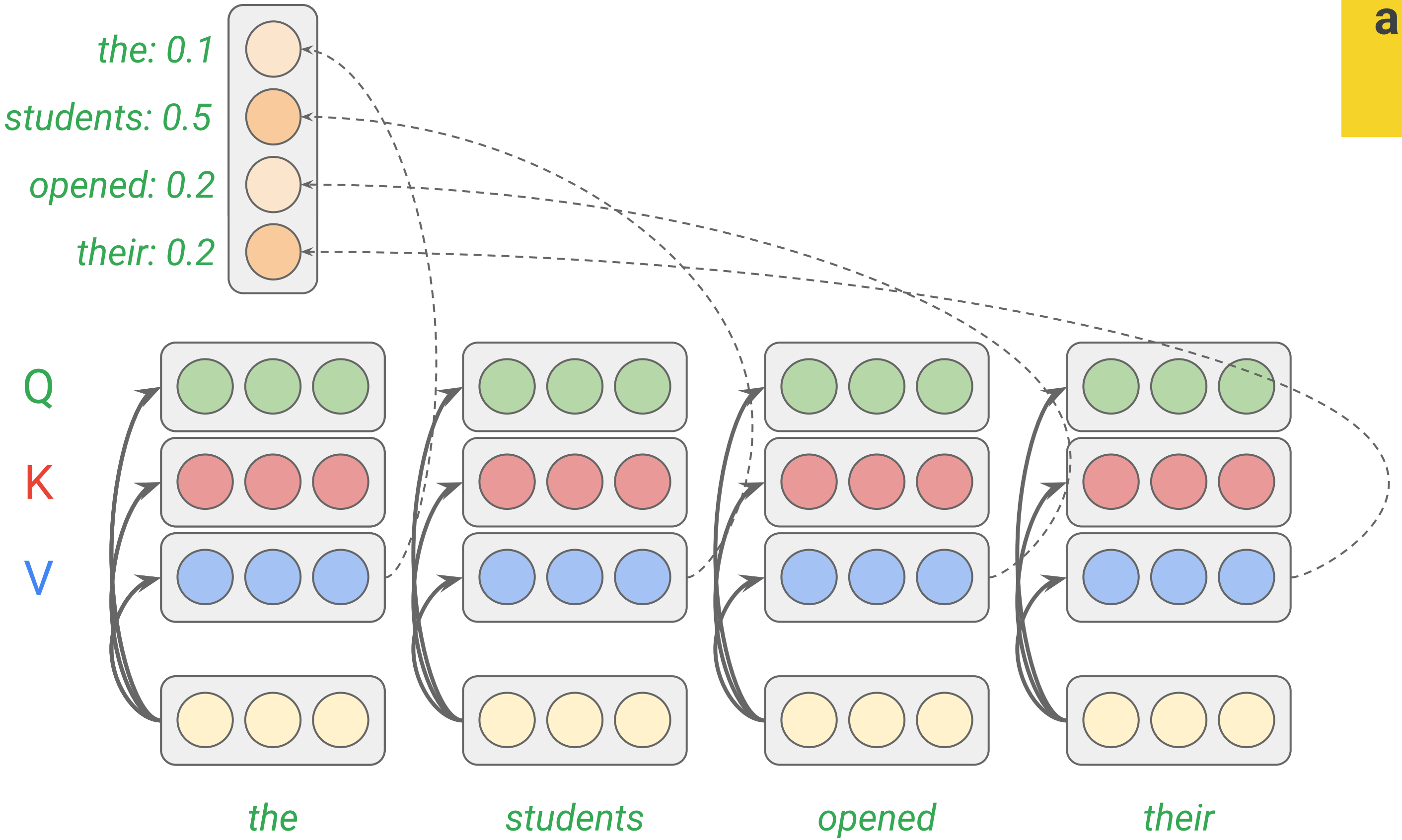
Attention mechanism review

all computations are parallelized

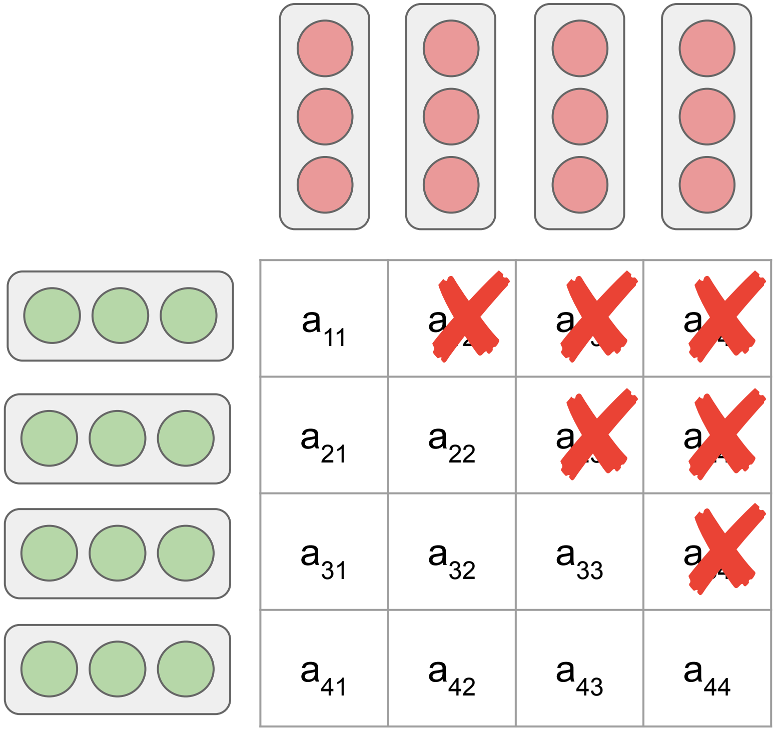


Attention mechanism review (cont'd)

***all* computations are parallelized**

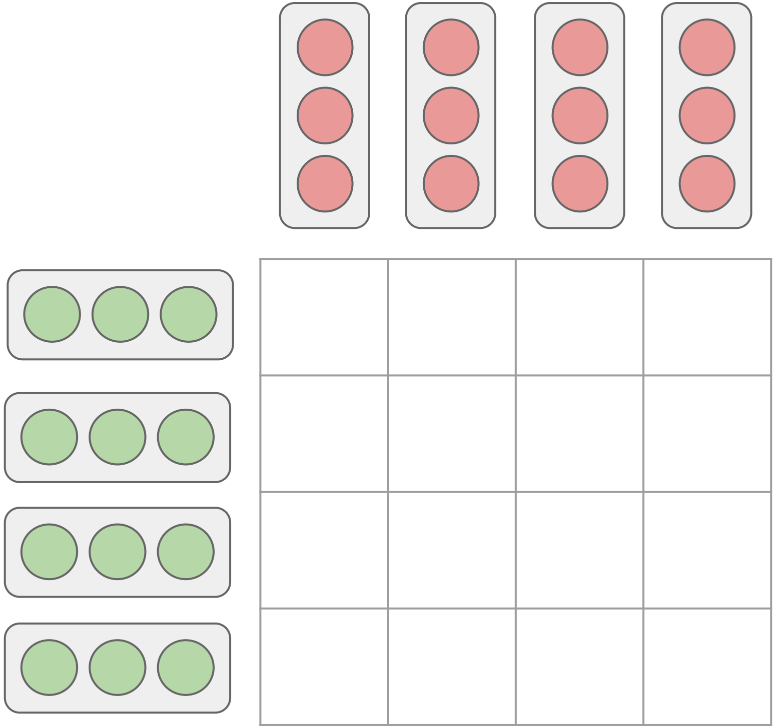


Attention mechanism review (cont'd)



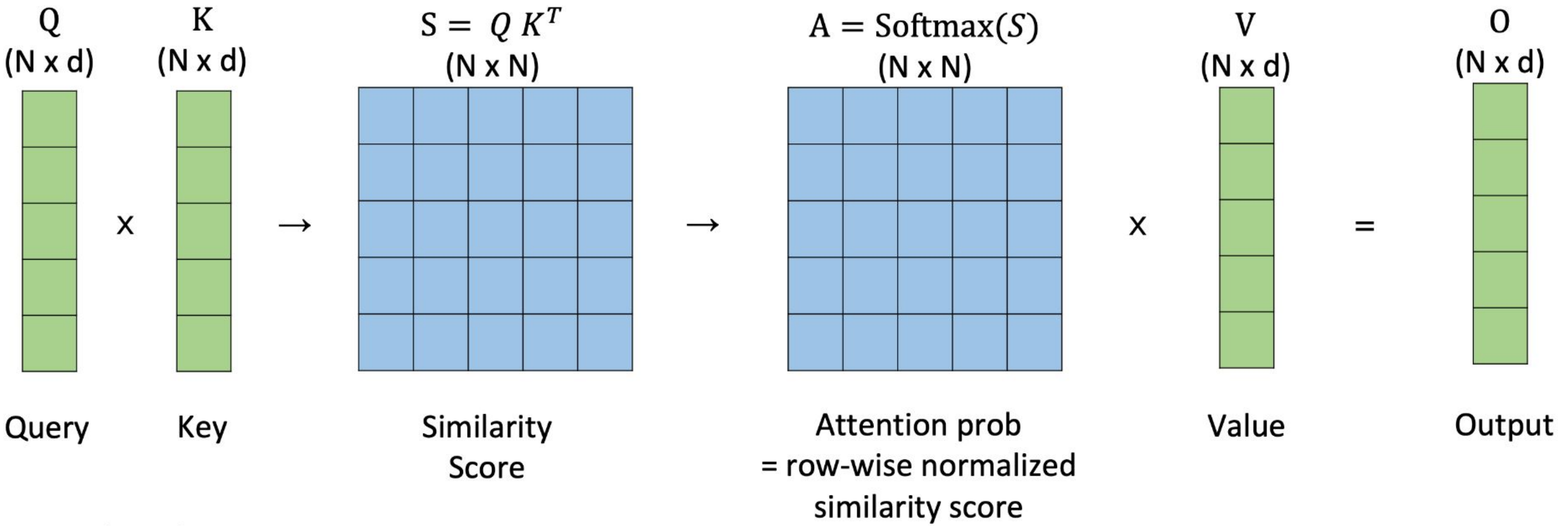
masking out all values in the input of the softmax which correspond to illegal connections

Quadratic complexity



The time complexity of self-attention is quadratic in the input length $O(n^2)$

Attention mechanism review (cont'd)



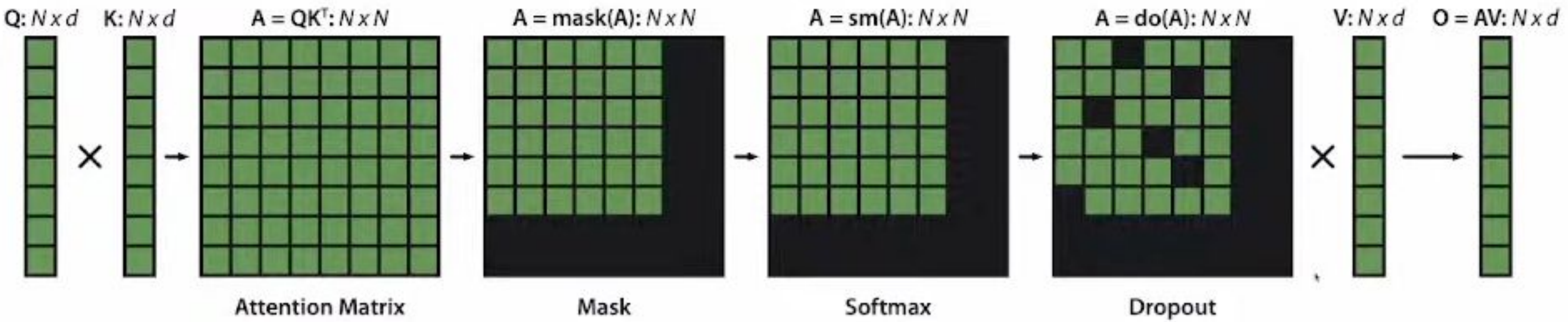
Typical sequence length N: 1K – 8K
Head dimension d: 64 – 128

$$\text{Softmax}([s_1, \cdots, s_N]) = \left[\frac{e^{s_1}}{\sum_i e^{s_i}}, \cdots, \frac{e^{s_N}}{\sum_i e^{s_i}}\right]$$

$$O = \text{Softmax}(QK^T)V$$

[FlashAttention – Tri Dao | Stanford MLSys #67]

Attention mechanism review (cont'd)



$$\mathbf{O} = \mathrm{Dropout}(\mathrm{Softmax}(\mathrm{Mask}(\mathbf{QK^T})))\mathbf{V}$$

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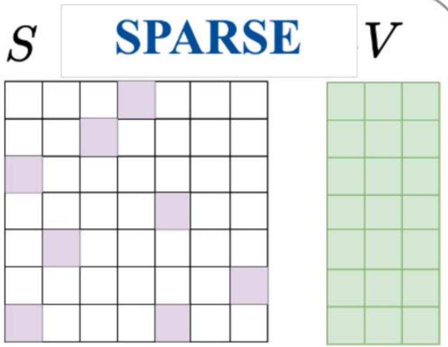
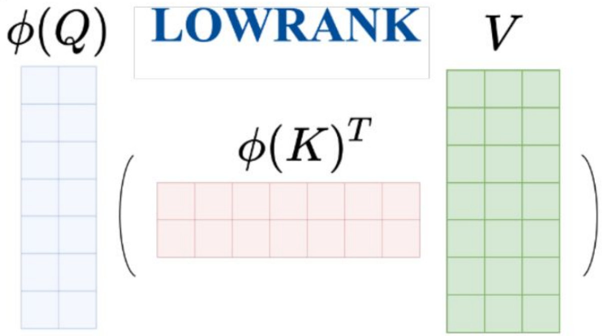
Approximate attention

tradeoff quality for ~~speed~~ fewer FLOPs

does not result in an actual wall clock speedup

Sparse Transformer
(Child et al. 19)
Reformer
(Kitaev et al. 20)
Routing Transformer
(Roy et al. 20)





Linformer
(Wang et al. 20)
Linear Transformer
(Katharopoulos et al. 20)
Performer
(Choromanski et al. 20)

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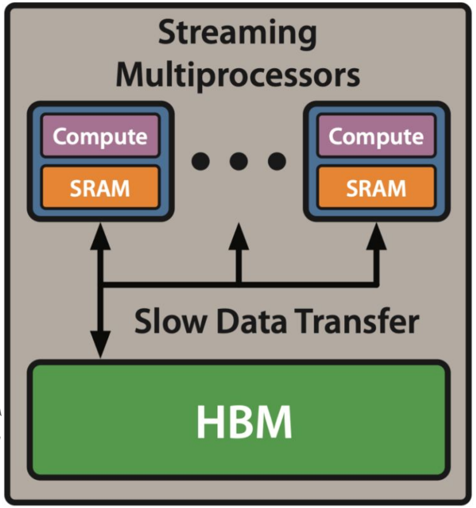
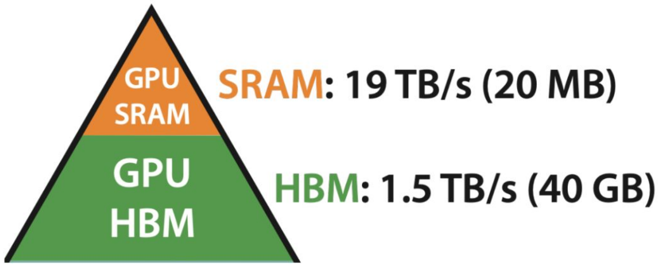
GPU compute model & memory hierarchy

2. Data moved to compute units & SRAM for computation

1. Inputs start out in HBM (GPU memory)

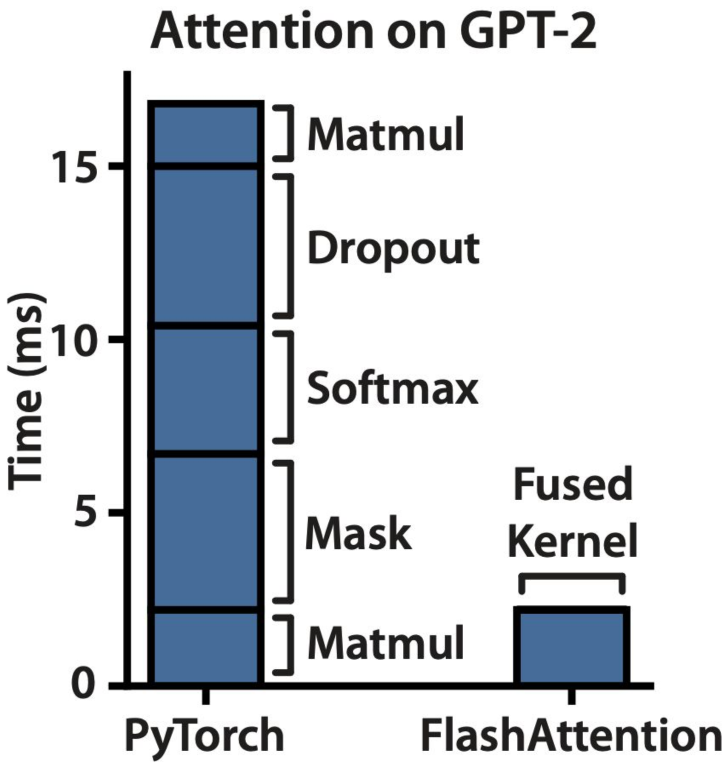
3. Output written back to HBM





Can we exploit the memory asymmetry to get speed up?

Data movement is the key bottleneck

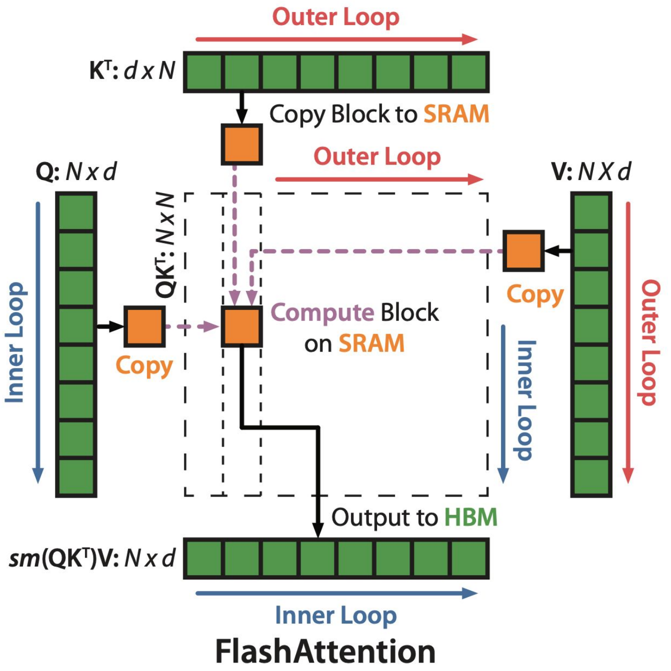


How to reduce HBM reads/writes: compute by blocks

- **Challenges:**
    - Compute softmax normalization without access to full input
    - Backward without the large attention matrix from forward
- **Approaches:**
    - **Tiling:** Restructure algorithm to load block by block from HBM to SRAM to compute attention
    - **Recomputation:** Don't store attention matrix from forward, recompute it in the backward

Tiling

- **Decomposing large softmax into smaller ones by scaling**



$$\mathrm{softmax}([A_1, A_2]) = [\alpha \times \mathrm{softmax}(A_1), \beta \times \mathrm{softmax}(A_2)]$$

$$\mathrm{softmax}([A_1, A_2]) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \alpha \times \mathrm{softmax}(A_1)V_1 + \beta \times \mathrm{softmax}(A_2)V_2$$

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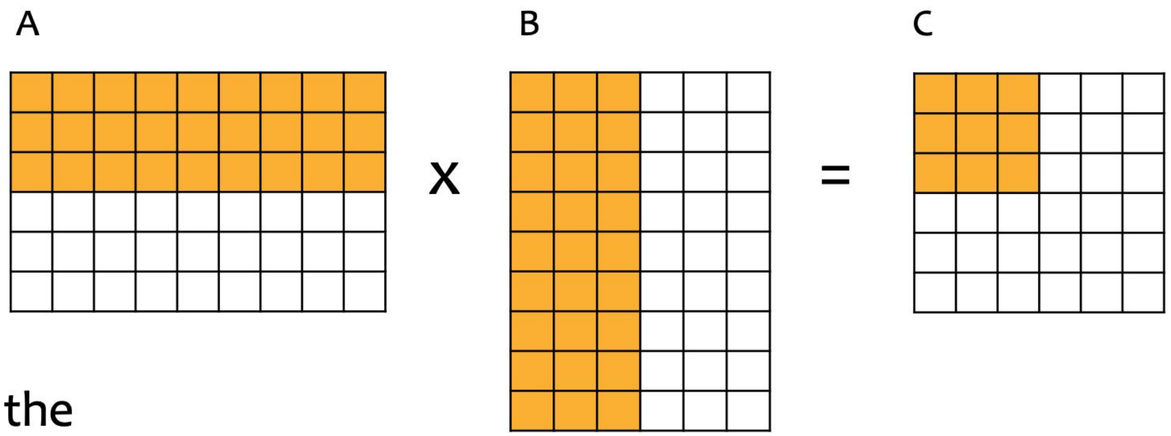
$$\mathrm{softmax}([a,b,c,d,e]) = \left[\frac{e^a}{e^a+e^b+e^c+e^d+e^e}, \frac{e^b}{e^a+e^b+e^c+e^d+e^e}, \frac{e^c}{e^a+e^b+e^c+e^d+e^e}, \frac{e^d}{e^a+e^b+e^c+e^d+e^e}, \frac{e^e}{e^a+e^b+e^c+e^d+e^e}\right]$$

$$\mathrm{softmax}([a,b,c,d,e]) = \left[\frac{e^a+e^b+e^c}{e^a+e^b+e^c+e^d+e^e} \cdot \left(\frac{e^a}{e^a+e^b+e^c}; \frac{e^b}{e^a+e^b+e^c}; \frac{e^c}{e^a+e^b+e^c}\right); \frac{e^d+e^e}{e^a+e^b+e^c+e^d+e^e} \cdot \left(\frac{e^d}{e^d+e^e}; \frac{e^e}{e^d+e^e}\right)\right]$$

; denotes concatenation
note that the terms involving $e^a + e^b + e^c$ cancel out each other
same for the $e^d + e^e$ terms

$$\mathrm{softmax}(\underbrace{[a,b,c,d,e]}_{A}) = \left[\underbrace{\frac{e^a+e^b+e^c}{e^a+e^b+e^c+e^d+e^e}}_{\alpha} \cdot \mathrm{softmax}(\underbrace{[a,b,c]}_{A_1}); \underbrace{\frac{e^d+e^e}{e^a+e^b+e^c+e^d+e^e}}_{\beta} \cdot \mathrm{softmax}(\underbrace{[d,e]}_{A_2})\right]$$

Tiling for matrix multiplication

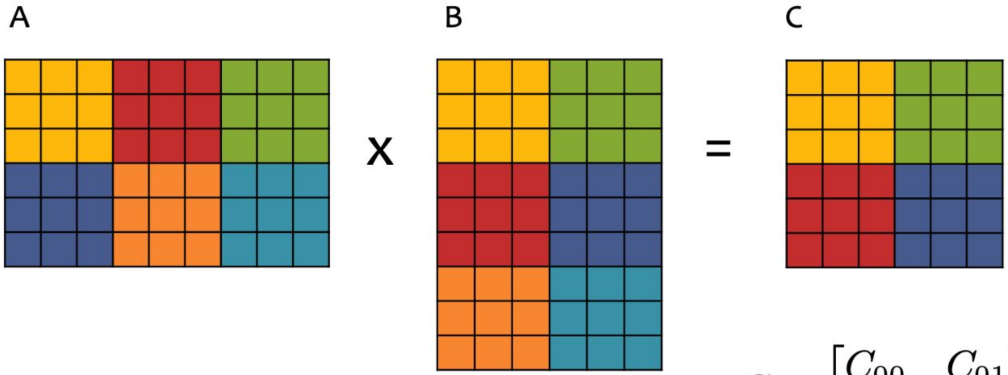


- We can view the computation as decomposing if we consider subsets of rows/columns

$$C_{(1,1):(3,3)} = A_{(1,1):(3,9)} \times B_{(1,1):(9,3)}$$

Tiling for matrix multiplication (cont'd)

- Tiling capitalizes on this decomposition
- Each output tile is computed by multiplying a pair of input tiles and adding it to the appropriate output tile



$$A = \begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \end{bmatrix}$$

with each $A_{ij} \in \mathbb{R}^{3\times 3}$

$$B = \begin{bmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \\ B_{20} & B_{21} \end{bmatrix}$$

with each $B_{ij} \in \mathbb{R}^{3\times 3}$

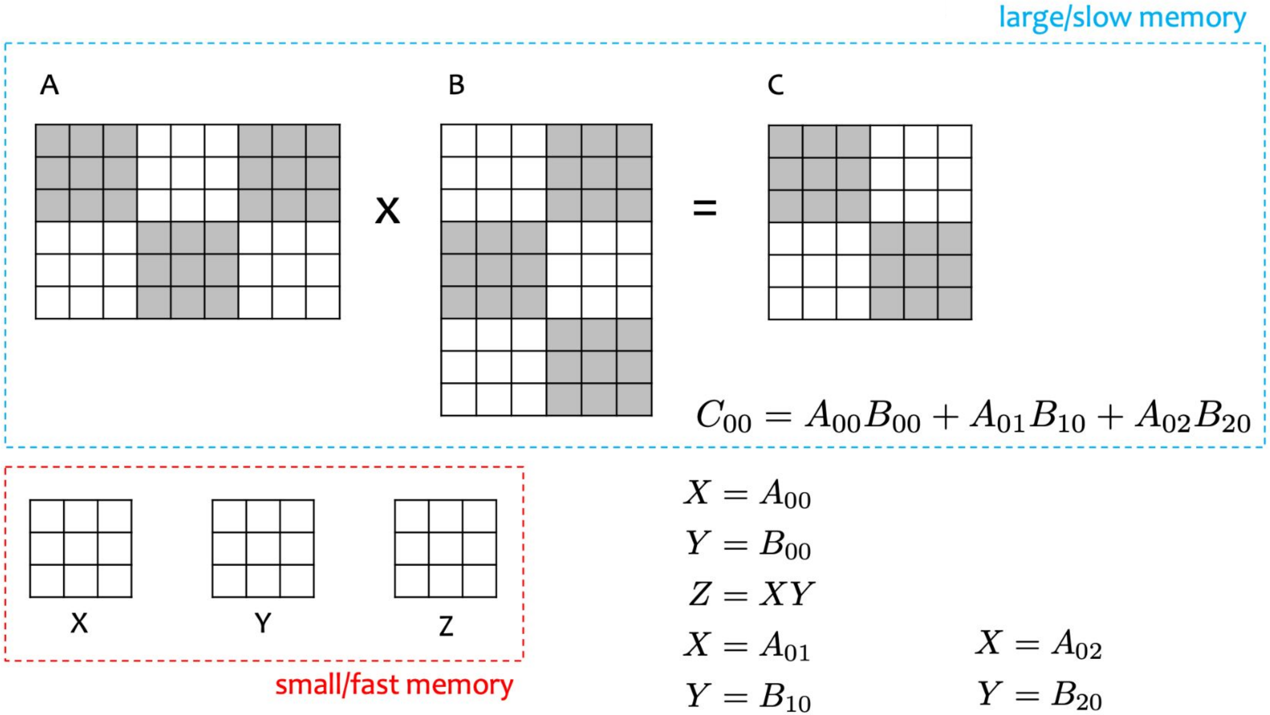
$$C = \begin{bmatrix} C_{00} & C_{01} \\ C_{10} & C_{11} \end{bmatrix}$$

with each $C_{ij} \in \mathbb{R}^{3\times 3}$

$$C_{00} = A_{00}B_{00} + A_{01}B_{10} + A_{02}B_{20}$$
$$C_{01} = A_{00}B_{01} + A_{01}B_{11} + A_{02}B_{21}$$
$$C_{10} = A_{10}B_{00} + A_{11}B_{10} + A_{12}B_{20}$$
$$C_{11} = A_{10}B_{01} + A_{11}B_{11} + A_{12}B_{21}$$

Tiling for matrix multiplication (cont'd)

- Tiling enables matrix multiplication of two **very** large matrices to capitalize on the small amount of fast memory on a device (e.g. GPU)
- Start by putting the input matrices and storage for the output matrix into large/slow memory
- Do the primary computation in slow/fast memory



$$C_{00} = A_{00}B_{00} + A_{01}B_{10} + A_{02}B_{20}$$

$$X = A_{00}$$
$$Y = B_{00}$$
$$Z = XY$$

$$X = A_{01}$$
$$Y = B_{10}$$
$$Z = Z + XY$$

$$X = A_{02}$$
$$Y = B_{20}$$
$$Z = Z + XY$$
$$C_{00} = Z$$

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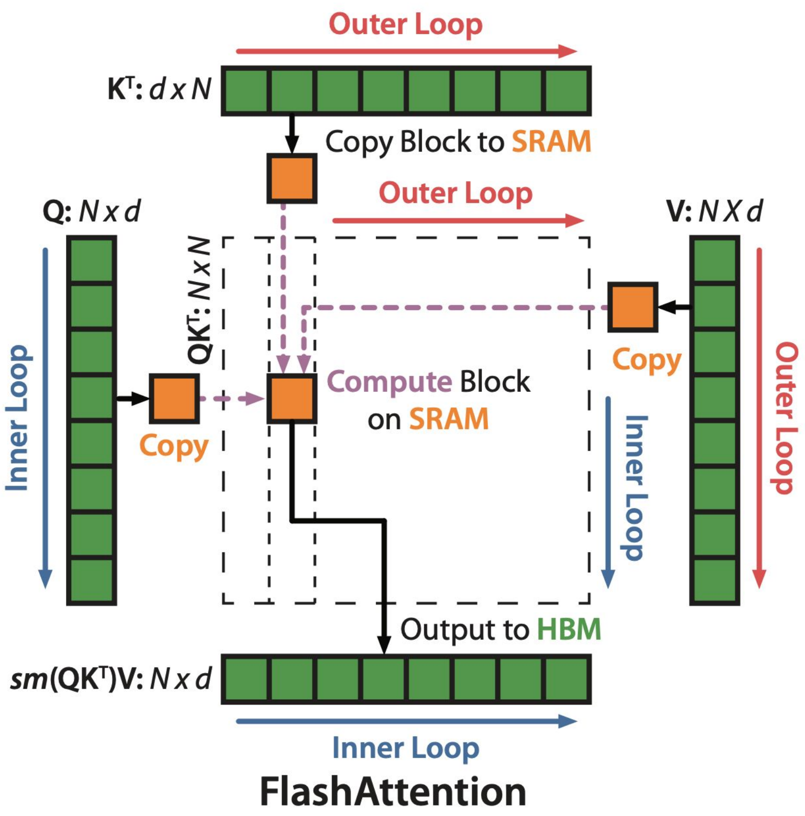
Tiling (cont'd)

1. Load inputs by blocks from HBM to SRAM.

2. On chip, compute attention output with respect to that block.

3. Update output in HBM by scaling.



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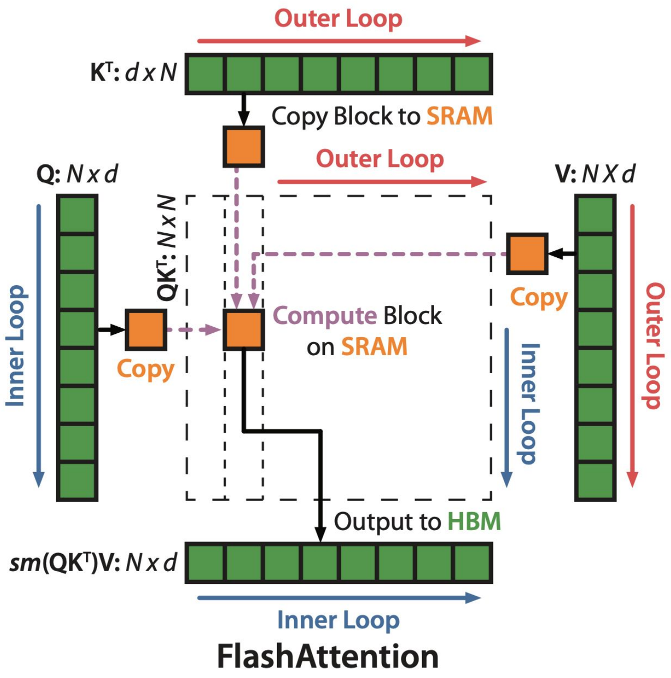
Demo

- https://jacksoncakes.com/flashattention-fast-and-memory-efficient-exact-attention/

Recomputation (backward pass)

- **By storing softmax normalization from forward (size N), quickly recompute attention in the backward from inputs in SRAM.**

Attention	Standard	FlashAttention
GFLOPs	66.6	75.2 (↑13%)
HBM reads/writes (GB)	40.3	4.4 (↓9x)
Runtime (ms)	41.7	7.3 (↓6x)



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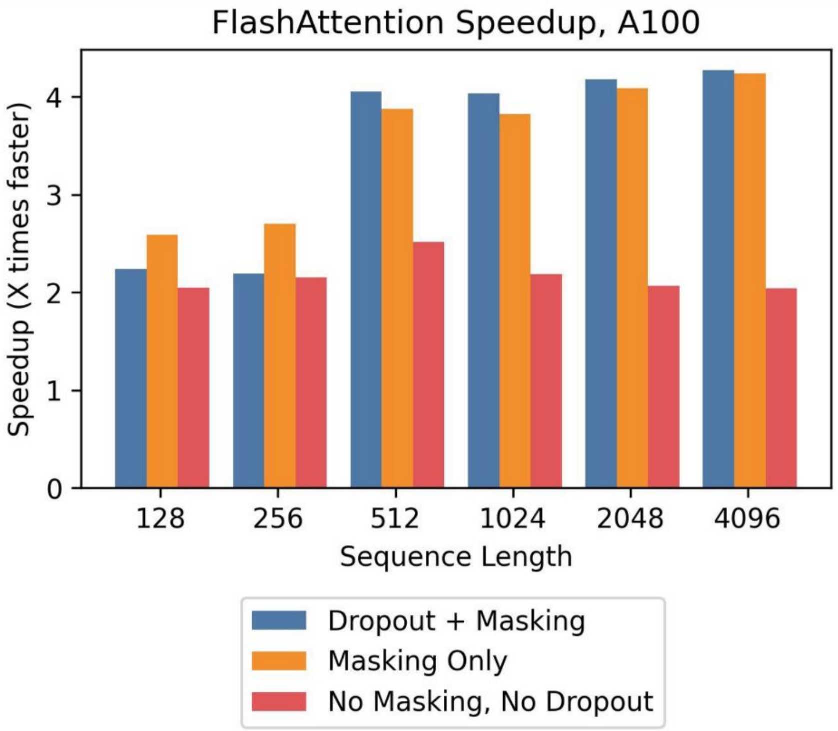
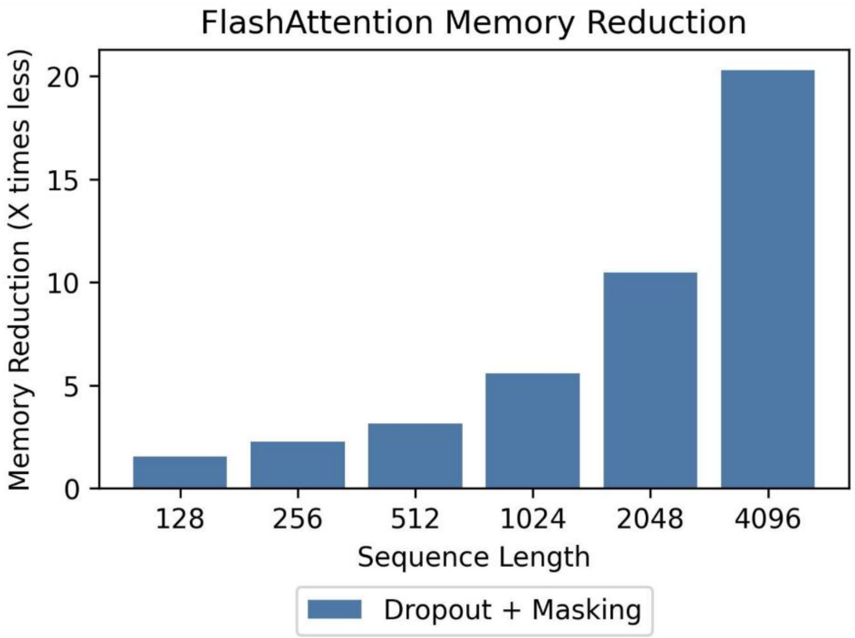
FlashAttention: 2-4x speedup, 10-20x memory reduction





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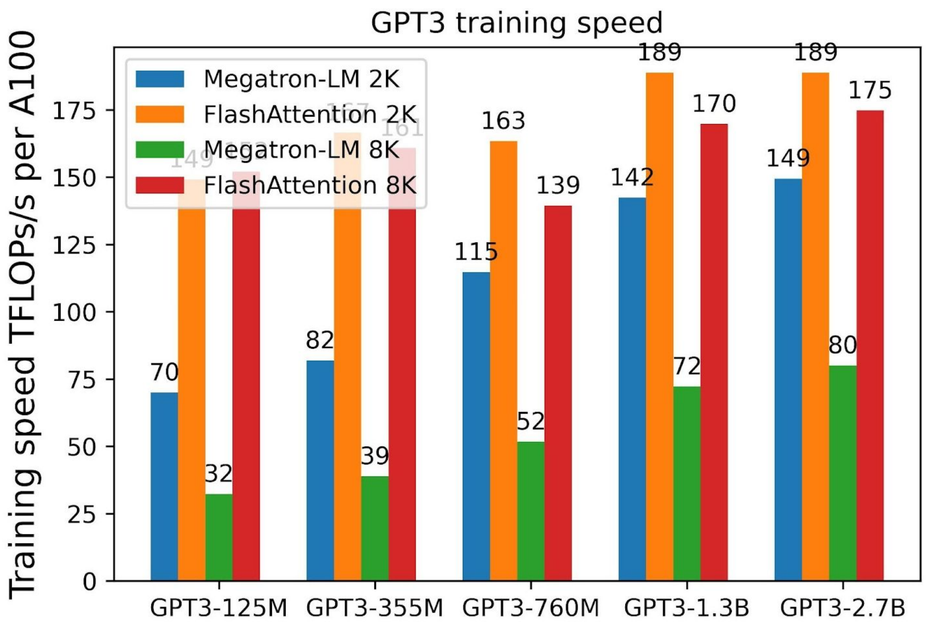
Faster Training: MLPerf Record for Training BERT-large

- MLPerf: (highly optimized) standard benchmark for training speed
- Time to hit an accuracy of 72.0% on MLM from a fixed checkpoint, averaged across 10 runs on 8 x A100 GPUs

BERT Implementation	Training time (minutes)
Nvidia MLPerf 1.1 [58]	20.0 ± 1.5
FlashAttention (ours)	$\mathbf{17.4} \pm 1.4$

Faster Training, longer context



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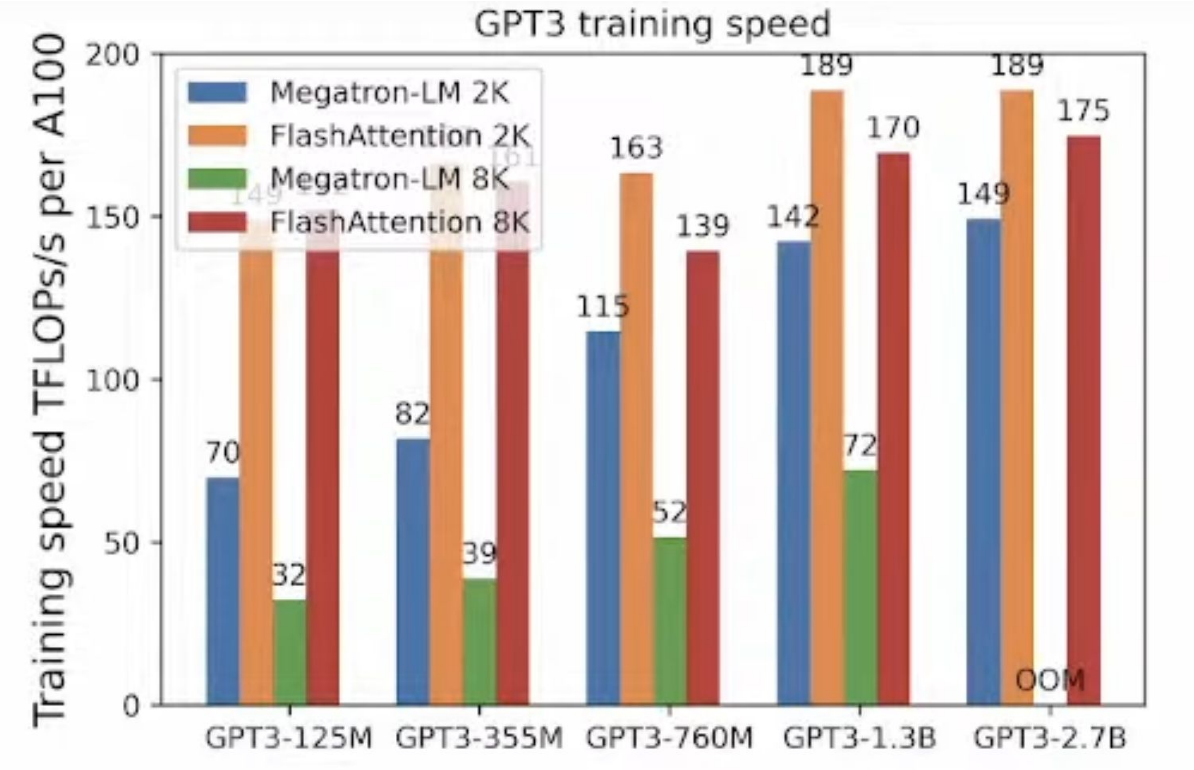
Faster Training, longer context



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Thank you!