Backpropagation

CS 5624: Natural Language Processing Spring 2025

https://tuvllms.github.io/nlp-spring-2025

Tu Vu



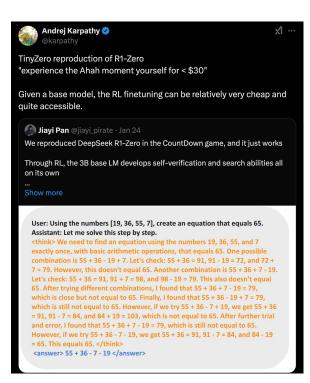
Logistics

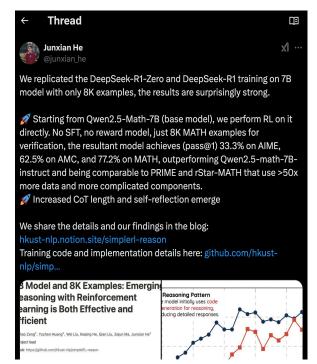
Final project group information due tomorrow 🚨

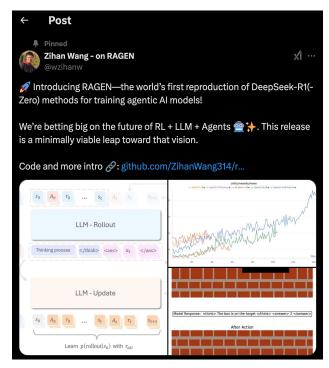


- Quiz0 will be released on Piazza tomorrow (due February
- HW0 was released on Piazza (due February 7th)

Efforts to replicate DeepSeek-R1







CountDown game

https://x.com/jiayi_pirate/status/ 1882839370505621655

Math

https://x.com/junxian_he/status/18 83183099787571519

LLM Agents

https://x.com/wzihanw/status/1884 092805598826609

SFT vs. RL

SFT Memorizes, RL Generalizes:

A Comparative Study of Foundation Model Post-training

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Abstract

Supervised fine-tuning (SFT) and reinforcement learning (RL) are widely used post-training techniques for foundation models. However, their respective role in enhancing model generalization remains unclear. This paper studies the comparative effect of SFT and RL on generalization and memorization, focusing on text-based and visual environments. We introduce GeneralPoints, an arithmetic reasoning card game, and also consider V-IRL, a real-world navigation environment, to assess how models trained with SFT

generalization (Bousquet & Elisseeff, 2000; Zhang et al., 2021) remain unclear, which makes it challenging to build reliable and robust AI systems. A key challenge in analyzing the generalization ability of foundation models (Bommasani et al., 2021; Brown et al., 2020) is separating data memorization¹ from the acquisition of transferable principles. We therefore investigate the key question of whether SFT or RL primarily memorize the training data (AllenZhu & Li, 2023a; Ye et al., 2024; Kang et al., 2024), or whether they learn generalizable principles that can adapt to novel task variants.

To address this question, we focus on two aspects of gener-

A recap on neural language models

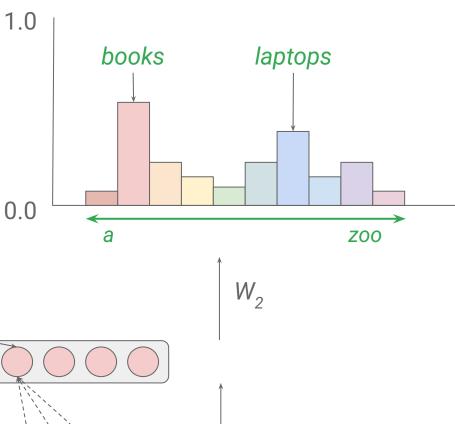
hidden layer

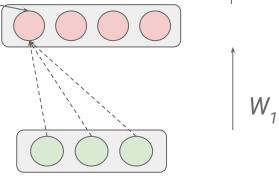
$$h = f(W_1 x)$$

hidden unit:
taking a weighted
sum of its inputs and
then applying a
non-linearity

h

X





Let
$$W=egin{bmatrix} w_{11}&w_{12}&w_{13}\ w_{21}&w_{22}&w_{23}\ w_{31}&w_{32}&w_{33}\ w_{41}&w_{42}&w_{43} \end{bmatrix}$$
 (dimensions $4 imes 3$) and $x=egin{bmatrix} x_1\ x_2\ x_3 \end{bmatrix}$ (dimensions $3 imes 1$).

Then, the multiplication yields the output vector h as:

$$\lceil h_1 \rceil \qquad \lceil w_{11}x_1 + w_{12}x_2 + w_{12}x_2 \rceil$$

$$h = egin{bmatrix} h_1 \ h_2 \ h_3 \ h_4 \end{bmatrix} = egin{bmatrix} w_{11}x_1 + w_{12}x_2 + w_{13}x_3 \ w_{21}x_1 + w_{22}x_2 + w_{23}x_3 \ w_{31}x_1 + w_{32}x_2 + w_{33}x_3 \ w_{41}x_1 + w_{42}x_2 + w_{43}x_3 \end{bmatrix}$$
 (dimensions $4 imes 1$).

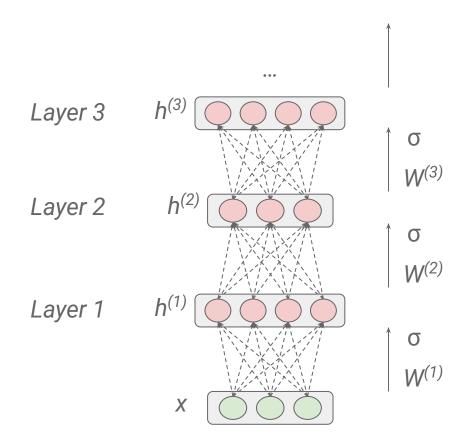
Let $W = egin{bmatrix} W_1 & W_2 & W_3 \end{bmatrix}$, where:

$$W_1=egin{bmatrix} w_{11}\w_{21}\w_{31}\w_{41} \end{bmatrix},\quad W_2=egin{bmatrix} w_{12}\w_{22}\w_{32}\w_{42} \end{bmatrix},\quad W_3=egin{bmatrix} w_{13}\w_{23}\w_{33}\w_{43} \end{bmatrix} \quad ext{(dimensions } 4 imes 1)$$
 and $x=egin{bmatrix} x_1\x_2\x_3 \end{bmatrix}$ (dimensions $3 imes 1$).

Then, the multiplication yields the output vector h as:

$$h = egin{bmatrix} h_1 \ h_2 \ h_3 \ h_4 \end{bmatrix} = W_1 x_1 + W_2 x_2 + W_3 x_3 = egin{bmatrix} w_{11} x_1 + w_{12} x_2 + w_{13} x_3 \ w_{21} x_1 + w_{22} x_2 + w_{23} x_3 \ w_{31} x_1 + w_{32} x_2 + w_{33} x_3 \ w_{41} x_1 + w_{42} x_2 + w_{43} x_3 \end{bmatrix}$$

Deep neural networks



hierarchical representations, where each layer builds upon the previous one

Bias values

$$h=\sigma(Wx+b)$$

Logits

Logits: the vector of scores right before the final softmax

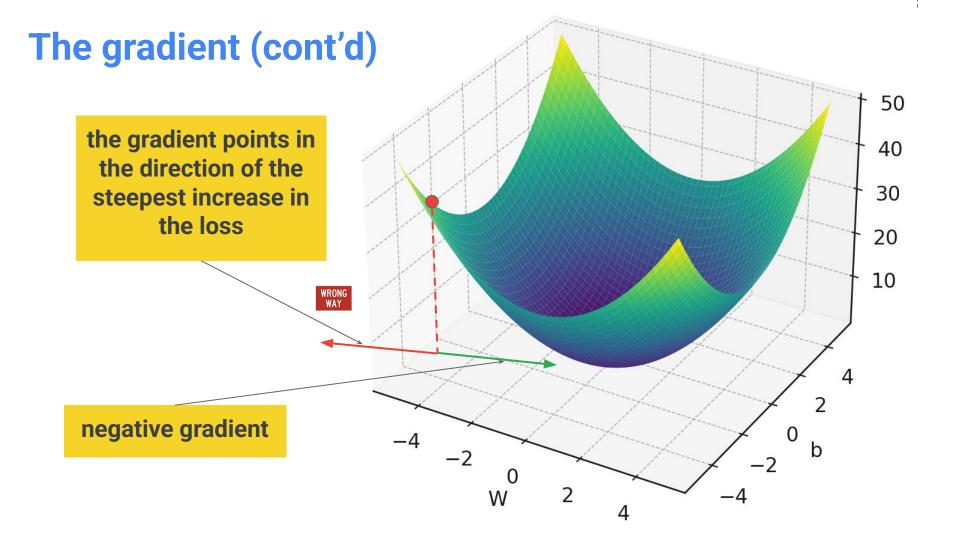
$$\hat{y} = egin{bmatrix} \hat{y}_1 \ \hat{y}_2 \ dots \ \hat{y}_V \end{bmatrix}$$

$$\hat{y}_i = rac{e^{z_i}}{\sum_{j=1}^{V} e^{z_j}}, \quad ext{for } i=1,2,\ldots,V$$

The partial derivative of the loss function

The partial derivative of the loss function L L with respect to the parameter w represents how much the loss changes as w W changes.



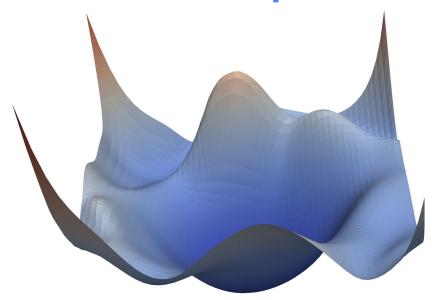


The loss landscape of neural nets

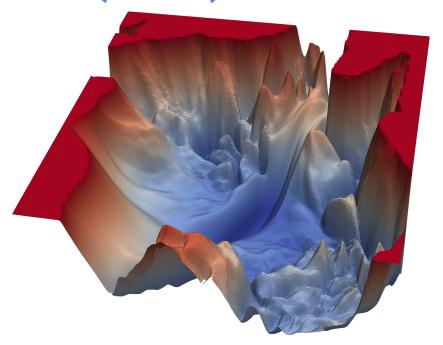
A convex function has at most one minimum; there are no local minima to get stuck in.

https://www.cs.umd.edu/~tomg/project/landscapes/

The loss landscape of neural nets (cont'd)

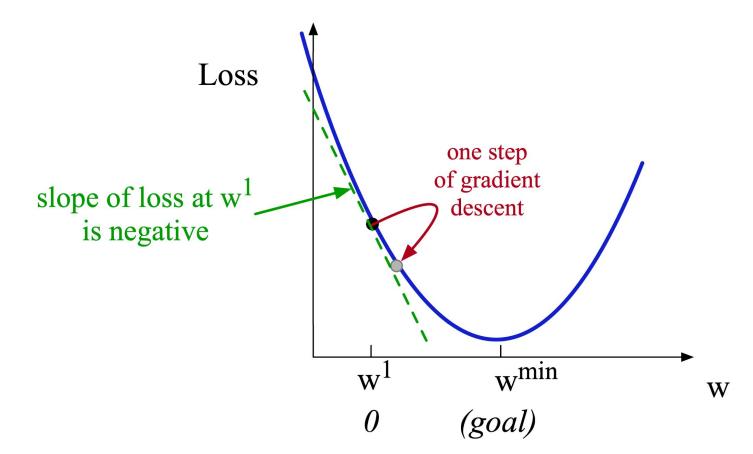


The loss for multi-layer neural networks is non-convex, and gradient descent may get stuck in local minima and never find the global optimum



https://www.cs.umd.edu/~tomg/project/ landscapes/

Gradient descent



Gradient descent (cont'd)

$$w_{t+1} = w_t - \eta \cdot rac{\partial L}{\partial w_t}$$

Where:

- w_t is the parameter at the current time step.
- ullet w_{t+1} is the updated parameter after applying the gradient.
- η is the learning rate, which controls the step size.
- $\frac{\partial L}{\partial w_t}$ is the gradient of the loss function L with respect to the parameter w_t , representing how the loss changes as the parameter changes.

Cross-entropy loss

The predicted probabilities

$$y = egin{bmatrix} y_1 \ y_2 \ dots \ y_V \end{bmatrix}$$

$$\hat{y} = egin{bmatrix} \hat{y}_1 \ \hat{y}_2 \ dots \ \hat{y}_V \end{bmatrix}$$

The ground truth label

1. if
$$i = c$$
 (correct class index

$$y_i = egin{cases} 1, & ext{if } i = c ext{ (correct class index)} \ 0, & ext{otherwise} \end{cases} \hat{y_i} = rac{e^{z_i}}{\sum_{j=1}^V e^{z_j}}, & ext{for } i = 1, 2, \dots, V$$

Cross-entropy loss (cont'd)

$$L_{CE}(\hat{y},y) = -\sum_{i=1}^V y_i \log \hat{y}_i$$

$$L_{CE}(\hat{y},y) = -\left(y_1 \log \hat{y}_1 + y_2 \log \hat{y}_2 + \dots + y_V \log \hat{y}_V
ight)$$

Since the true label y is one-hot encoded, only one term in the sum is nonzero, corresponding to the correct class c, where $y_c=1$ and $y_i=0$ for all $i\neq c$. This simplifies the sum to:

$$L_{CE}(\hat{y},y) = -y_c \log \hat{y}_c$$

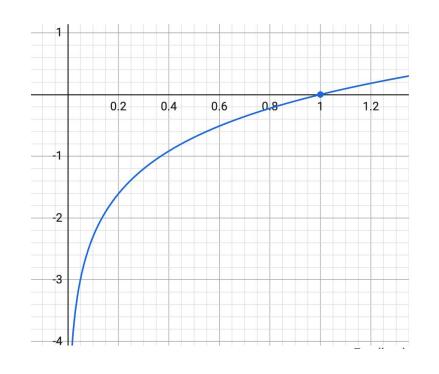
Since $y_c = 1$, this further reduces to:

$$L_{CE}(\hat{y},y) = -\log \hat{y}_c$$

Cross-entropy loss (cont'd)

$$L_{CE}(\hat{y},y) = -\log \hat{y}_c$$

$$\hat{y}_c
ightarrow 0$$
, $\log \hat{y}_c
ightarrow -\infty$



- If $\hat{y}_c = 0.9$, then $\log(0.9) pprox -0.105$, and the loss will be small.
- If $\hat{y}_c = 0.1$, then $\log(0.1) pprox -2.302$, and the loss will be much larger.

Backpropagation

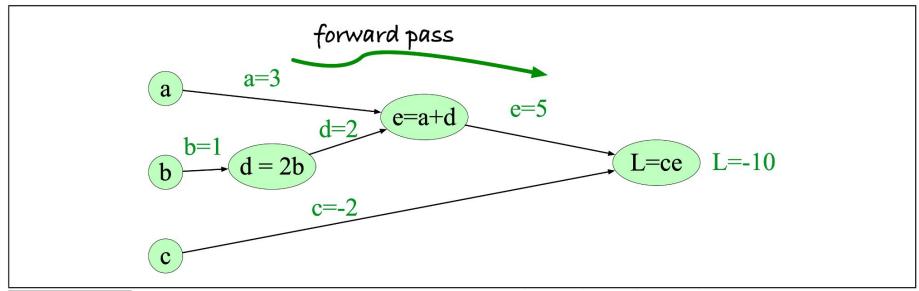


Figure 7.12 Computation graph for the function L(a,b,c) = c(a+2b), with values for input nodes $a=3,\,b=1,\,c=-2$, showing the forward pass computation of L.

Backpropagation (cont'd)

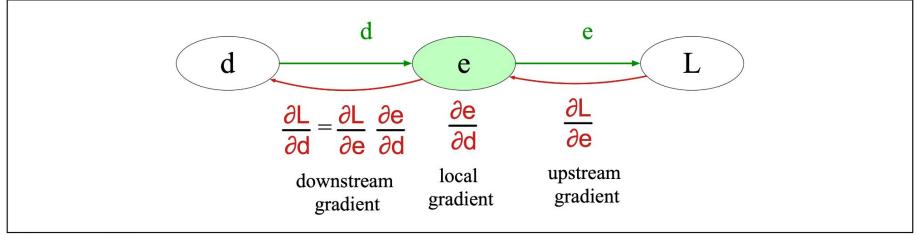


Figure 7.13 Each node (like *e* here) takes an upstream gradient, multiplies it by the local gradient (the gradient of its output with respect to its input), and uses the chain rule to compute a downstream gradient to be passed on to a prior node. A node may have multiple local gradients if it has multiple inputs.

Backpropagation (cont'd)

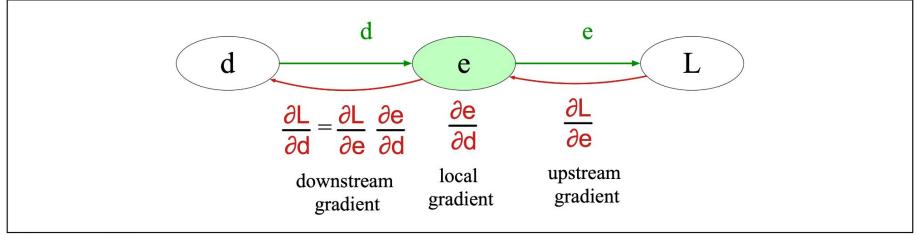


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Updating parameters

$$w_{t+1} = w_t - \eta \cdot rac{\partial L}{\partial w_t}$$

Thank you!